Localization spectrum of a bath-coupled generalized Aubry-André model in the presence of interactions

Yi-Ting Tu (涂懿庭)

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BACKGROUND

Nonergodicity vs locality

Parameter

Nonergodicity vs locality

Nonergodicity vs locality

Ergodic
(Satisfies ETH) ⟷ Transition ⟷ Non-ergodic
(Violates ETH)

→ Parameter

Extended
(Volume law EE) ⟷ Transition ⟷ Localized
(Area law EE)

Nonergodicity vs locality

Ergodic
(Satisfies ETH)

Transition ⟷

Non-ergodic
(Violates ETH)

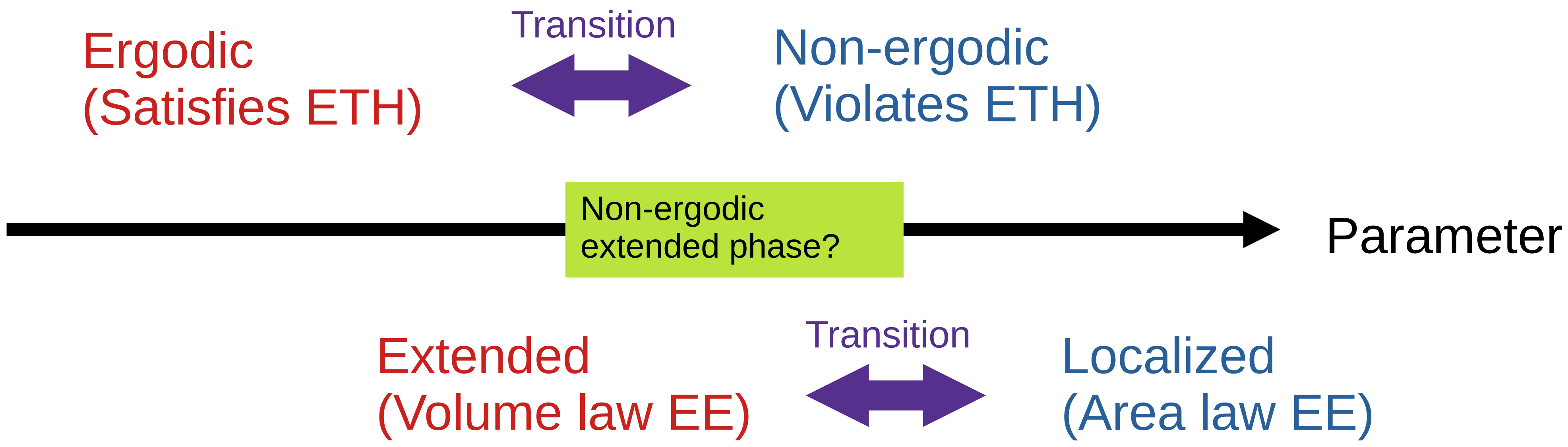
→ Parameter

Extended
(Volume law EE)

Transition ⟷

Localized
(Area law EE)

Nonergodicity vs locality



Interacting Ganeshan-Pixley-Das Sarma (GPD) model

$$H = \sum_{j=1}^{L-1} \left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + V S_j^z S_{j+1}^z \right) + W \sum_{j=1}^{L} h_j S_j^z$$

$$h_j = \frac{\cos(2\pi\varphi j + \phi)}{1 - \alpha \cos(2\pi\varphi j + \phi)} \qquad \varphi = \frac{1 + \sqrt{5}}{2}$$

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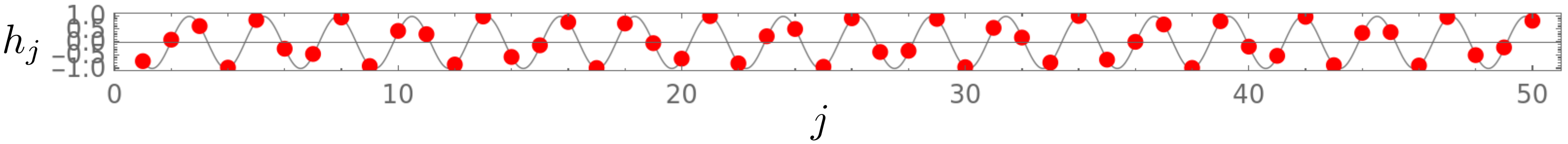
$\alpha = 0$

Aubry-André (AA) model



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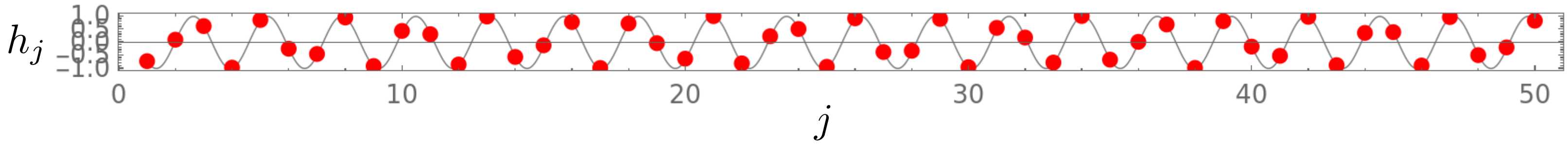
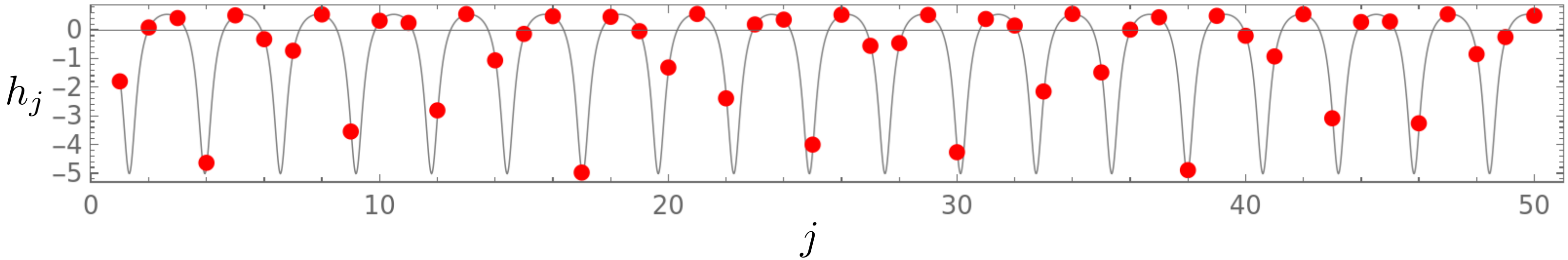


$\alpha = -0.8$



Evidence of NEE phase in the interacting GPD model

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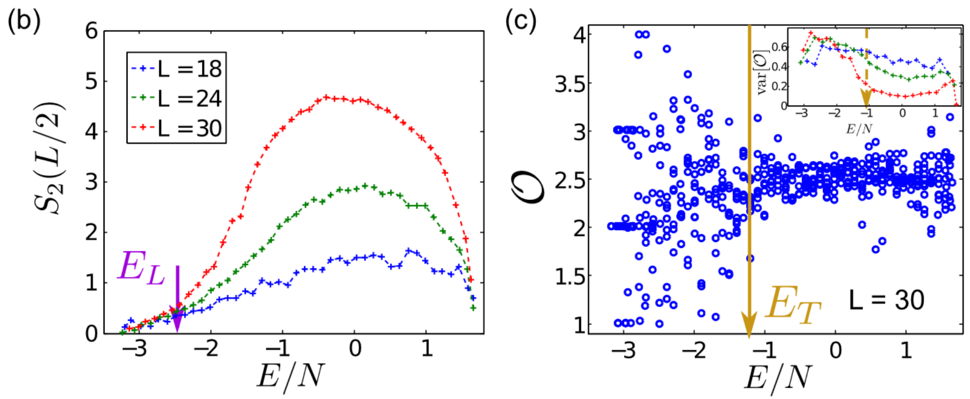

Depends on the finite size behaviors of **two very different quantities**.

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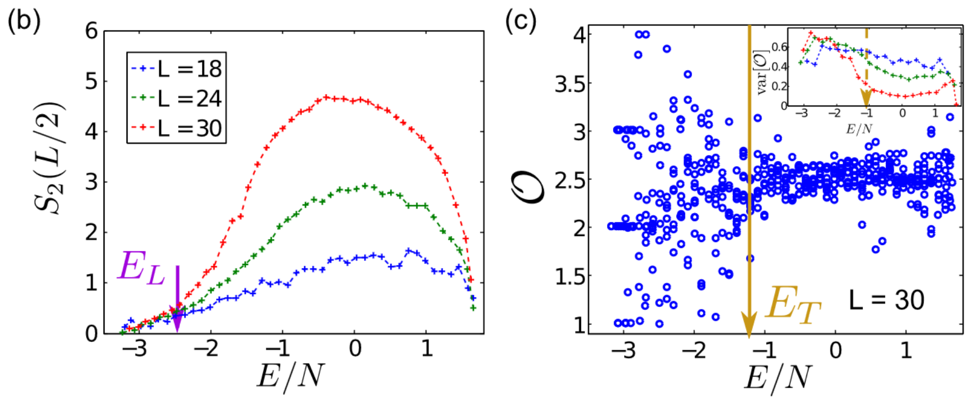

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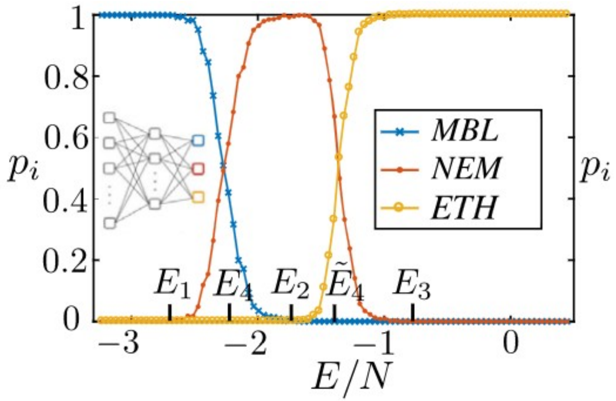

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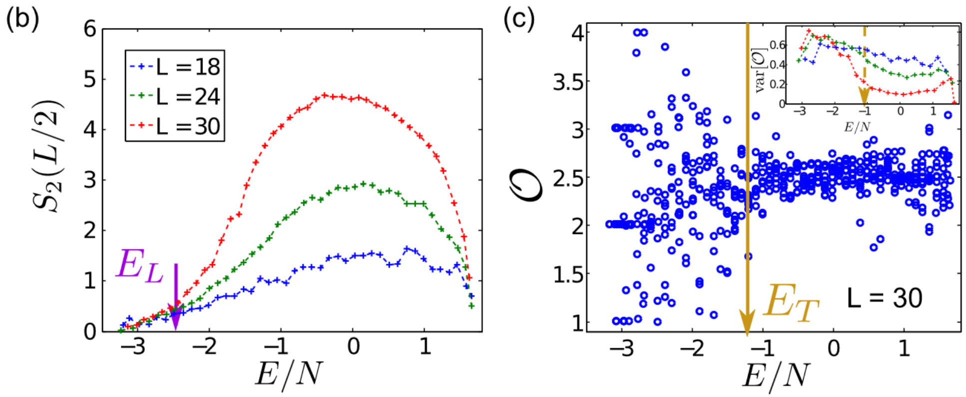

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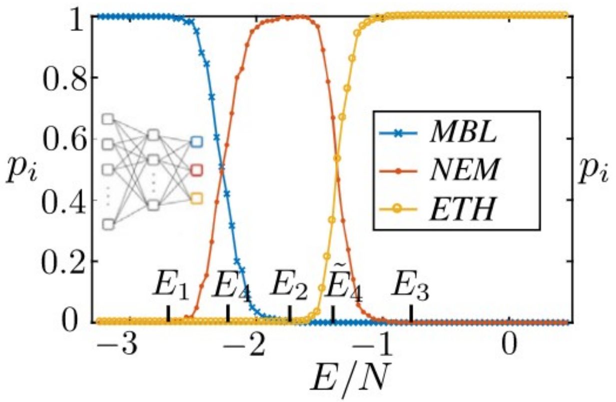

Uses a **very large feature space** which is difficult to interpret.

Can we use a single quantity to generate a 2D feature space?

(function of time)

OUR SETUP

System

$$H_{\text{system}} = \sum_{j=1}^{L_s-1} \left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + V S_j^z S_{j+1}^z\right) + W \sum_{j=1}^{L_s} h_j S_j^z$$

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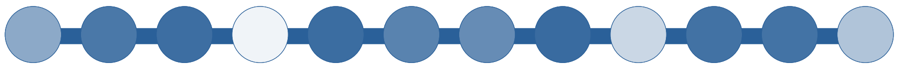

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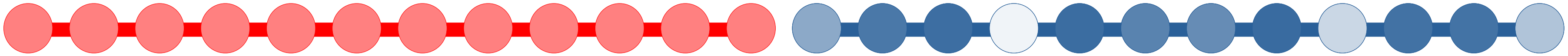

$$H_{\text{bath}} = \sum_{j=1-L_b}^{-1} \left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + S_j^z S_{j+1}^z\right)$$

System

Size (=12)

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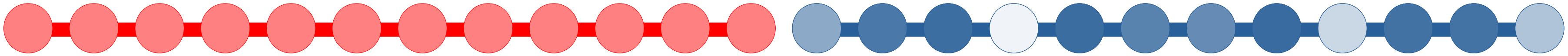

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Disorder strength (scanned from 0.1 to 3.9)

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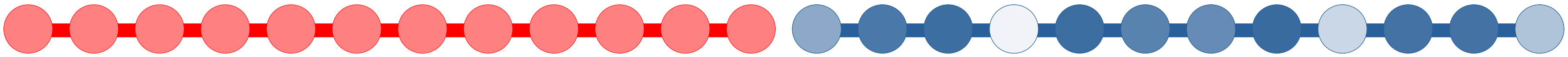

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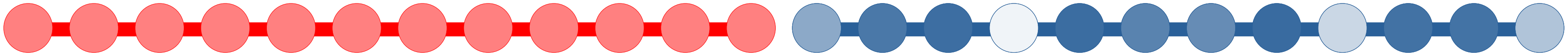
Initial phase (averaged over 14 random choices)



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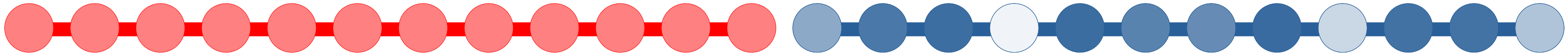
Size (=12)

Dynamics



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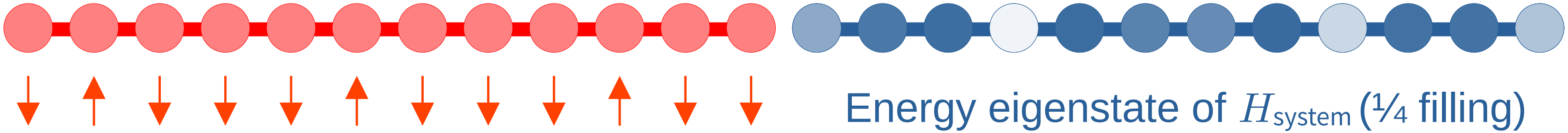
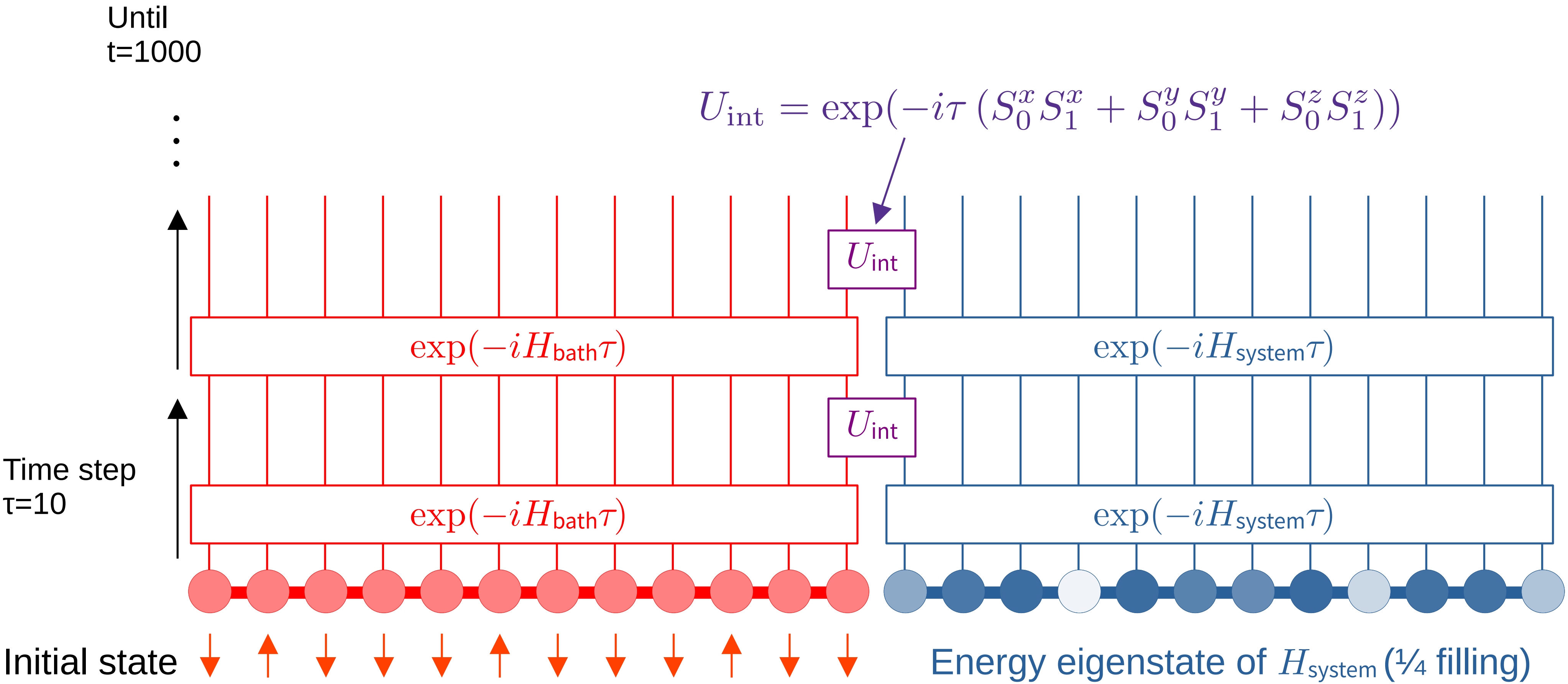


Initial state

Energy eigenstate of H_{system} (¼ filling)

Dynamics



Until
t=1000
$U_{\text{int}} = \exp(-i\tau\,(S_0^x S_1^x + S_0^y S_1^y + S_0^z S_1^z))$
U_{int}
$\exp(-iH_{\text{bath}}\tau)$
$\exp(-iH_{\text{system}}\tau)$
U_{int}
Time step
τ=10
$\exp(-iH_{\text{bath}}\tau)$
$\exp(-iH_{\text{system}}\tau)$
Initial state
Energy eigenstate of H_{system} (¼ filling)

Growth of entanglement entropy

Entanglement entropy between system and bath

$$\tilde{S}(t) = \frac{S_{\max} - S(t)}{S_{\max}}$$

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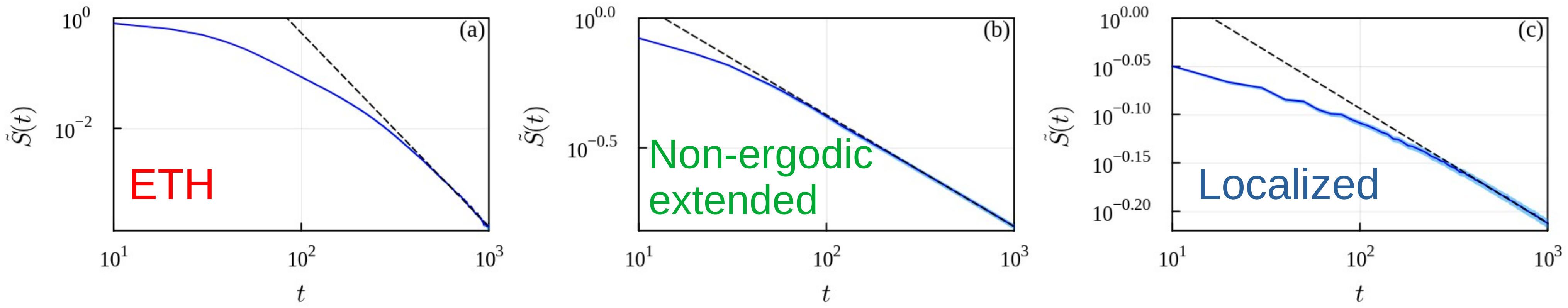
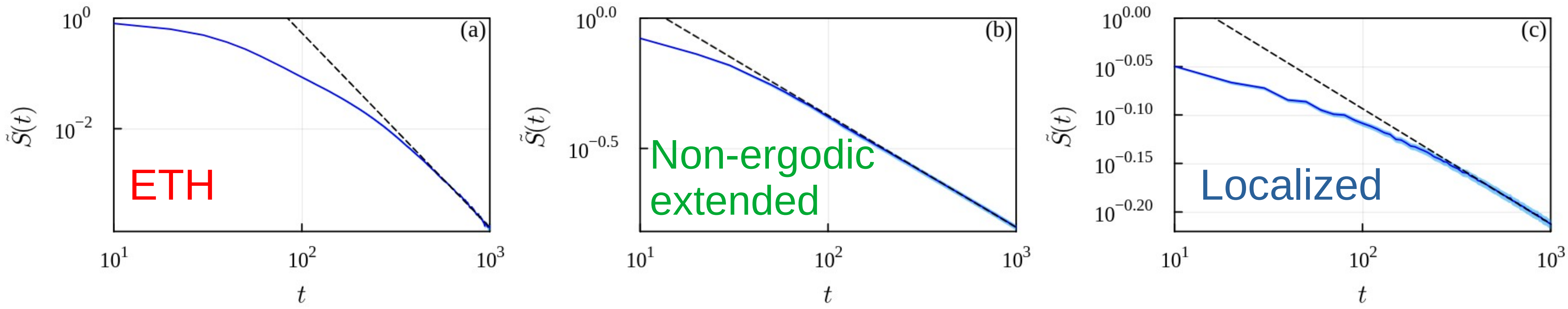


Growth of entanglement entropy

Entanglement entropy between system and bath

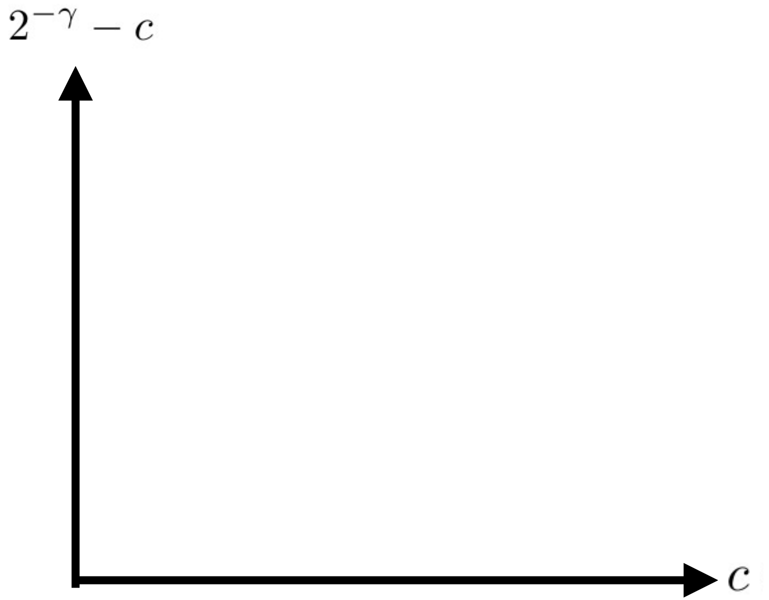
$$\tilde{S}(t) = \frac{S_{\max} - S(t)}{S_{\max}} \approx c\left(\frac{t}{t_0}\right)^{-\gamma}$$

from t_0=500 to t_1=1000



THREE-PHASE CLASSIFICATION

The feature space



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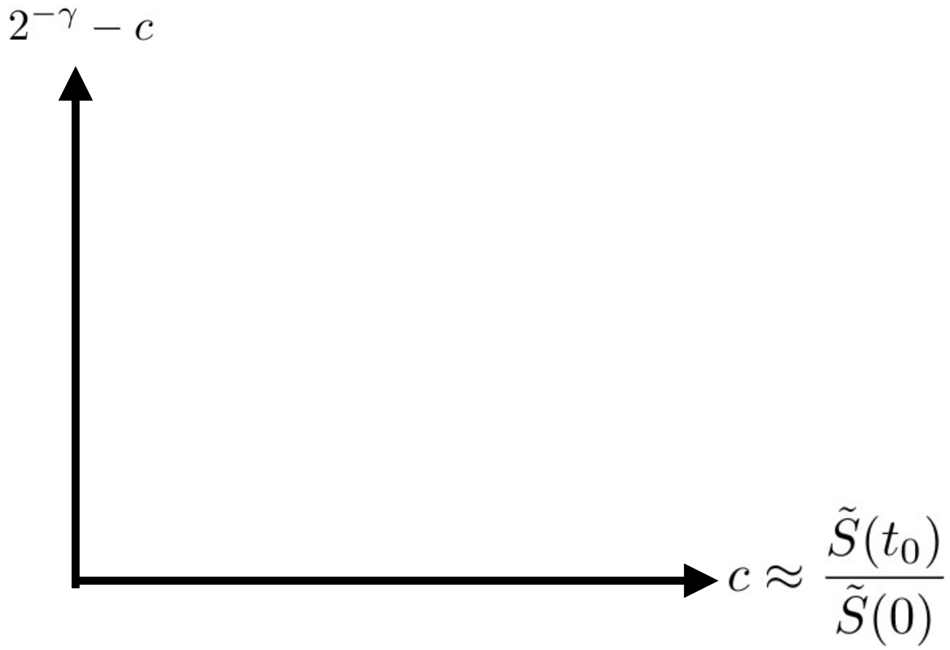
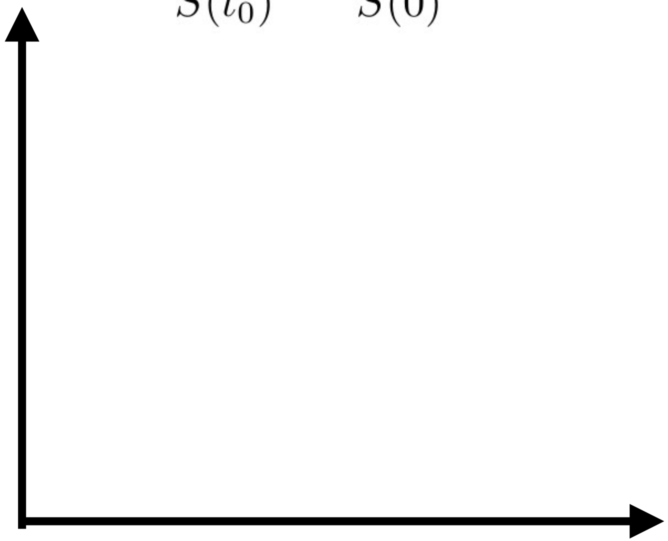


The feature space

The feature space

$$2^{-\gamma} - c \approx \frac{\tilde{S}(2t_0)}{\tilde{S}(t_0)} - \frac{\tilde{S}(t_0)}{\tilde{S}(0)}$$



$$c \approx \frac{\tilde{S}(t_0)}{\tilde{S}(0)}$$

(early-time slowness)

The feature space

(late-time deceleration)

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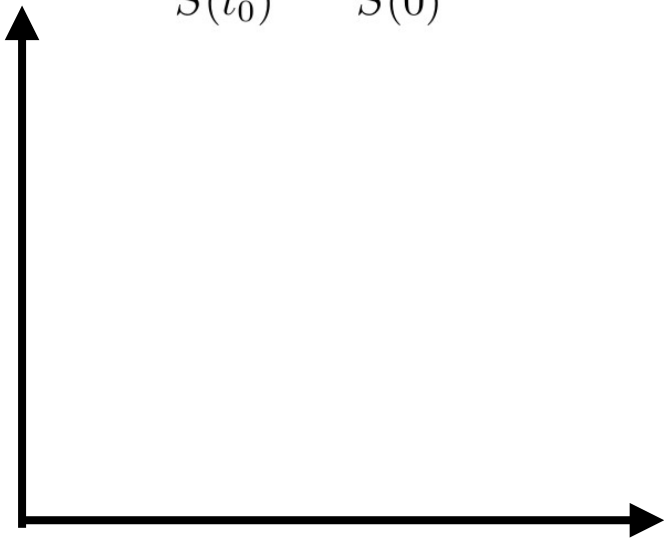

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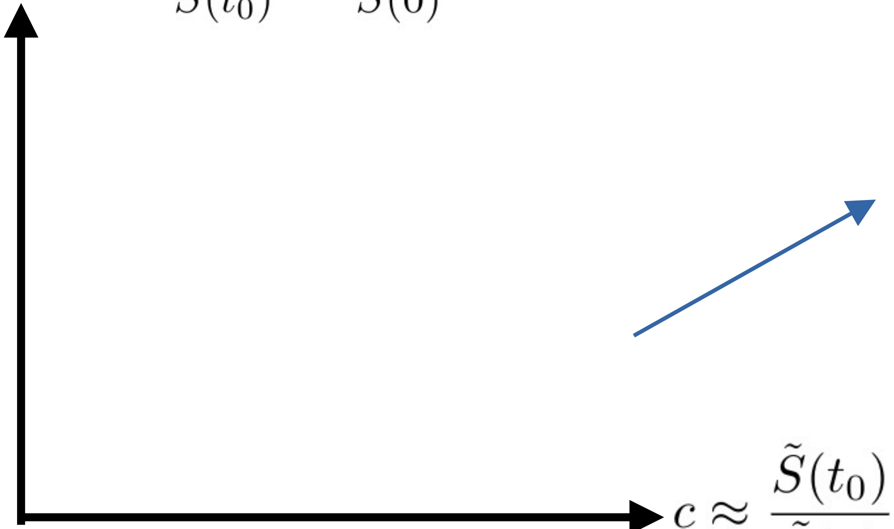

Thermalization is slow at all time.
(no initial fast-hybridization)

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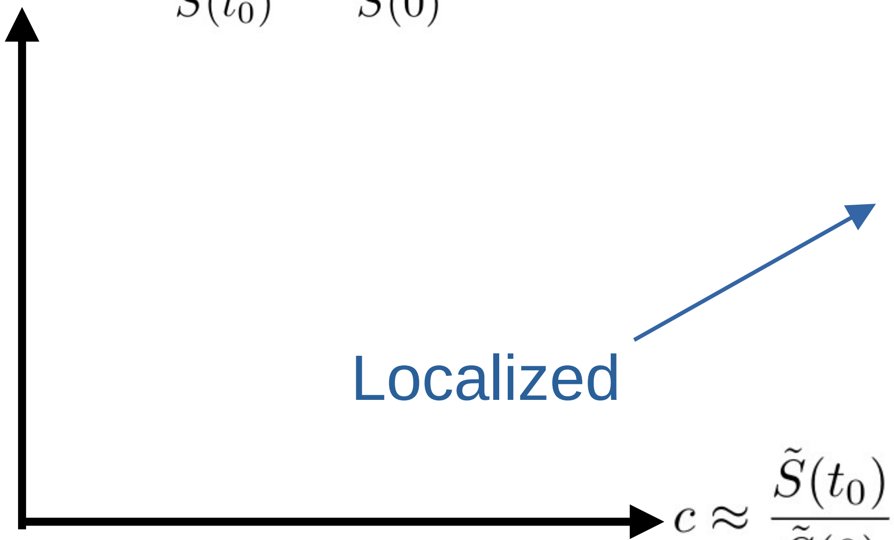

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Fast initially
(bath hybridizes with
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Slow at late time
(no seed to thermalize
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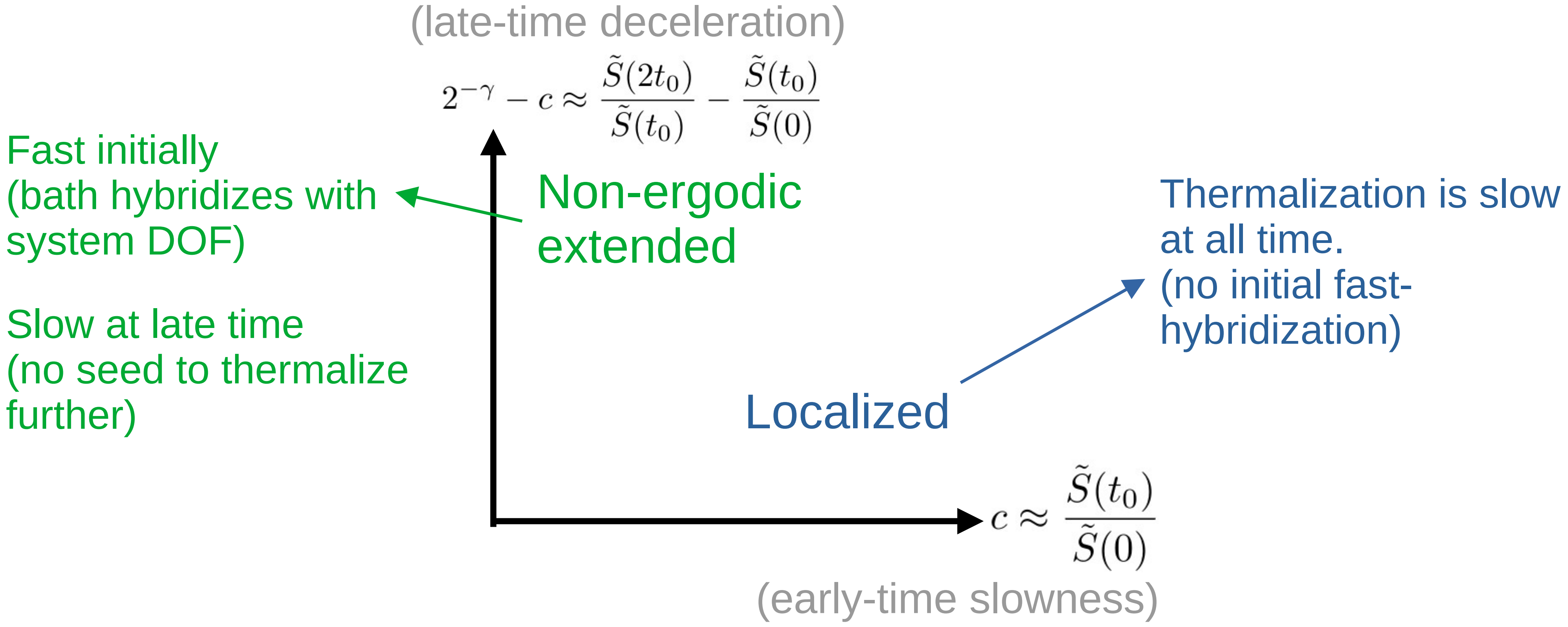
The feature space



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Non-ergodic extended

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Fast at all time

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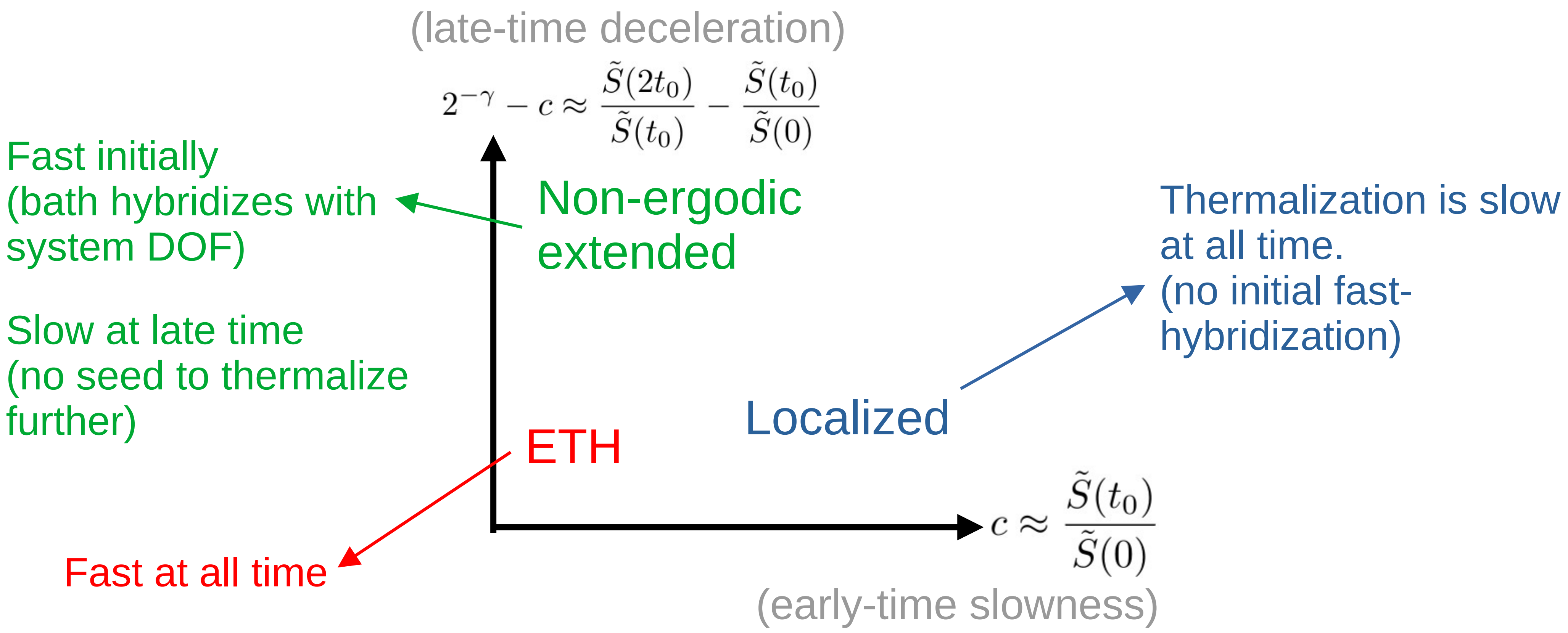
(early-time slowness)

The feature space



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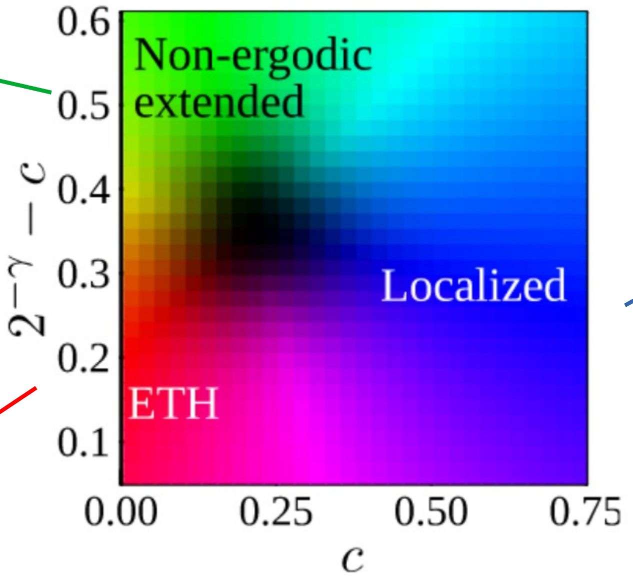

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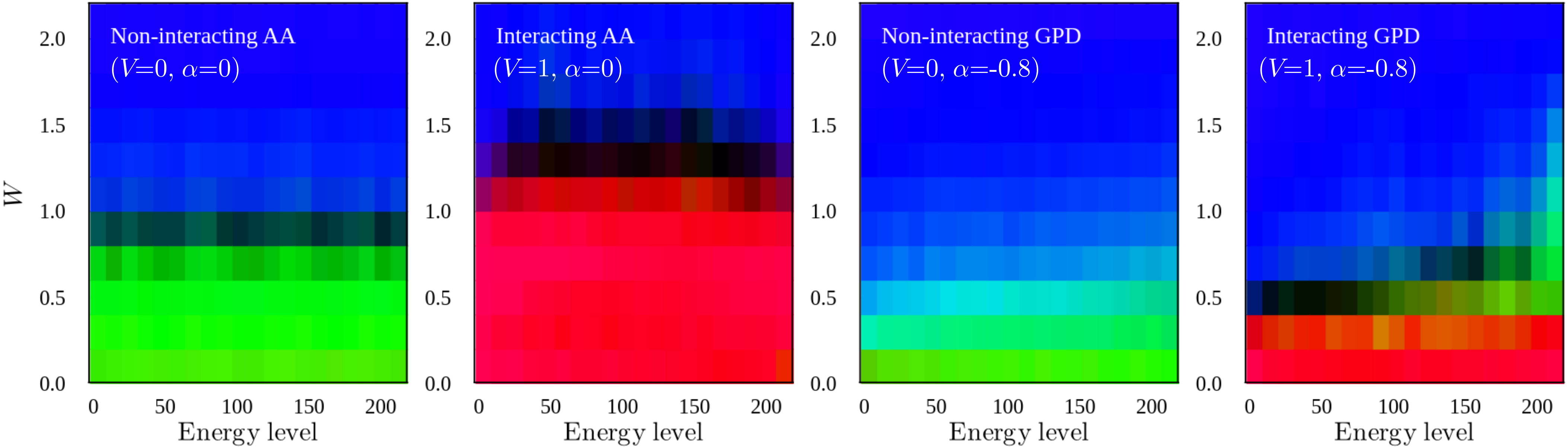
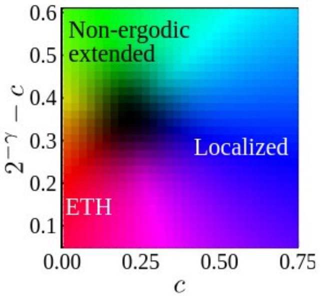
Fast at all time

Results – three-phase classification



Color map:



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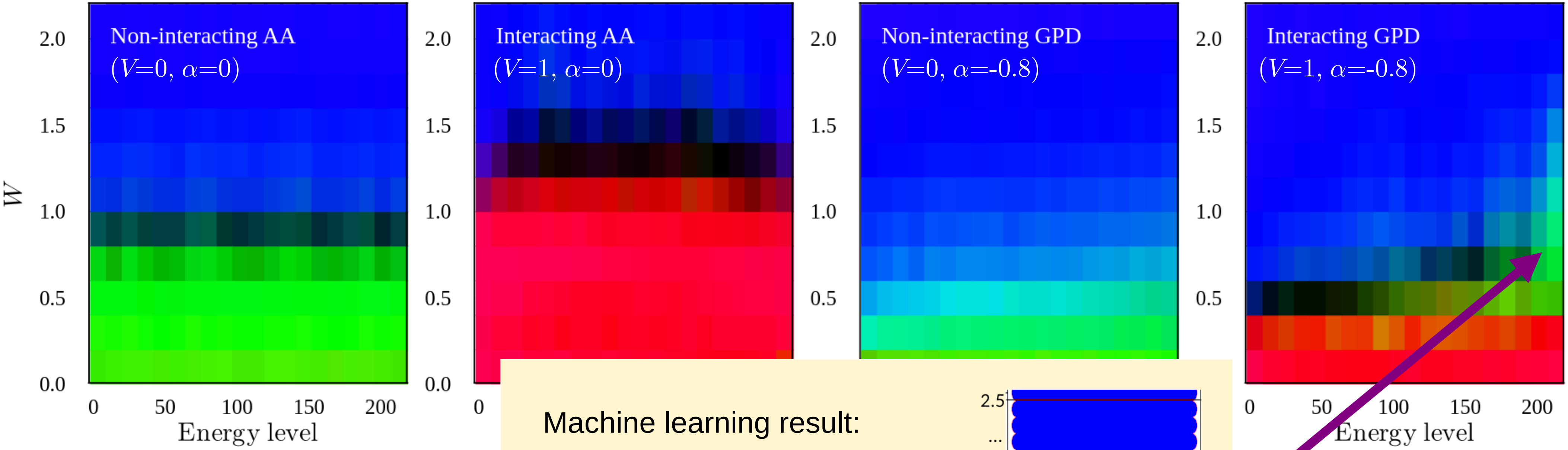

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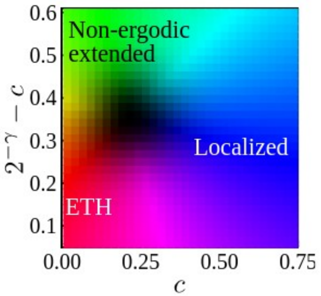
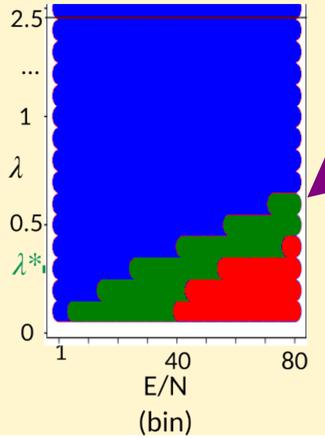


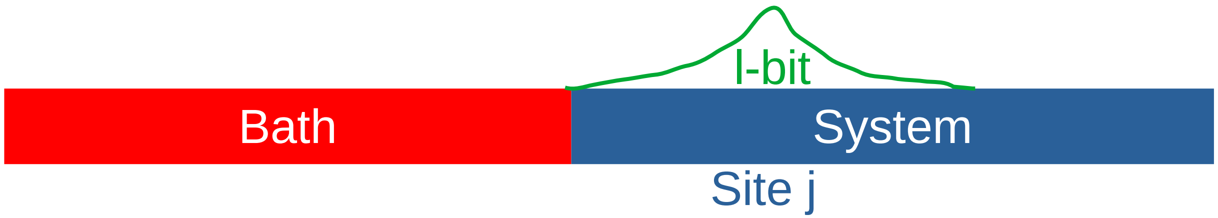
Machine learning result:

Beveridge, Rodrigo, Li,
Barbierato, Hsu, to be published



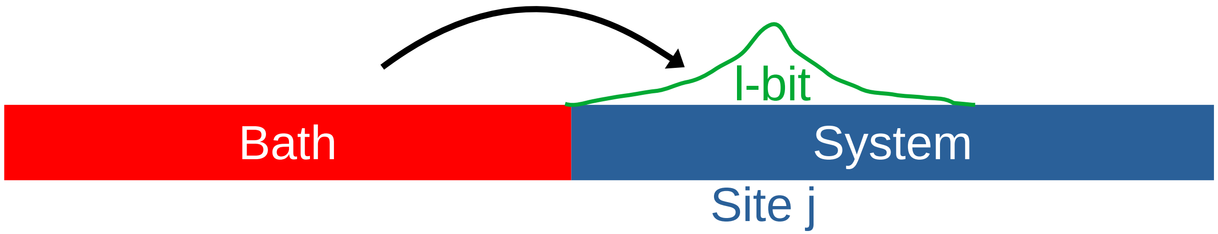
LOCALIZATION LENGTH IN INTERMEDIATE
AND
STRONG DISORDER REGIMES

Extraction of localization lengths



Extraction of localization lengths

Coupling strength $g_j = g_0 e^{-\frac{j}{\xi_{\text{loc}}}}$



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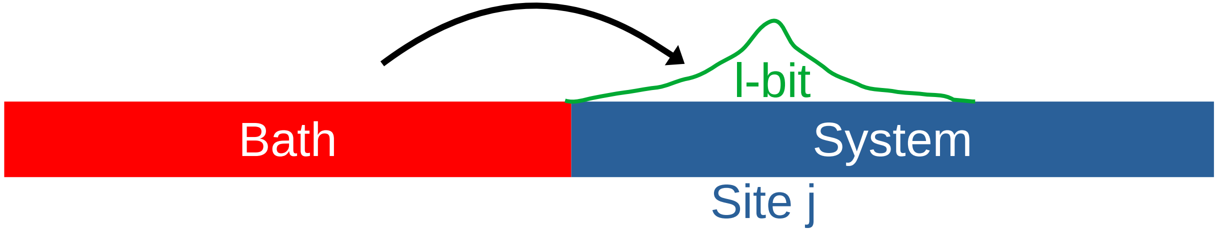
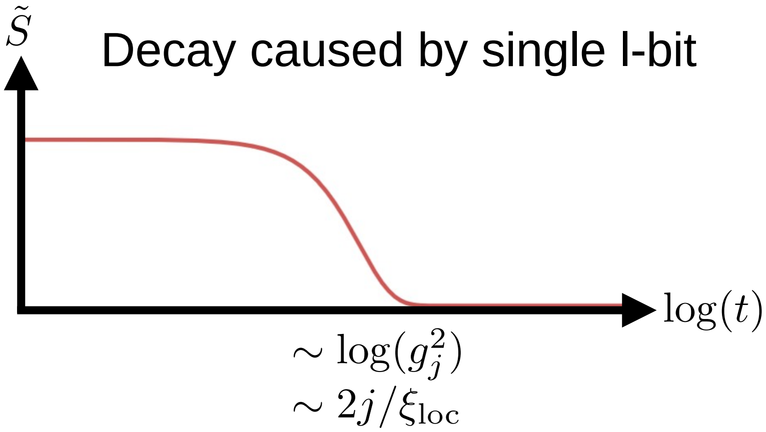


Decay caused by single l-bit



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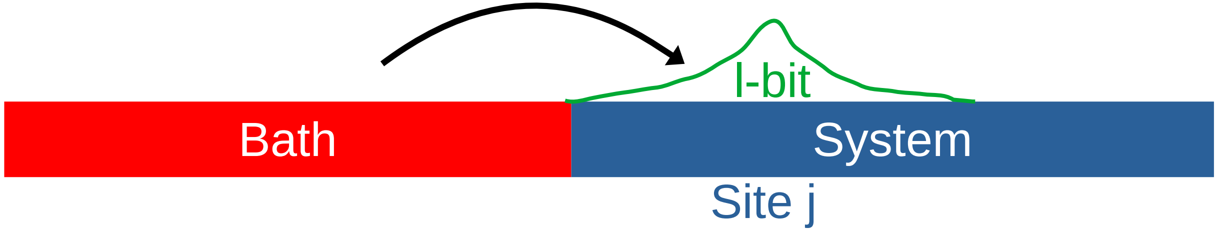

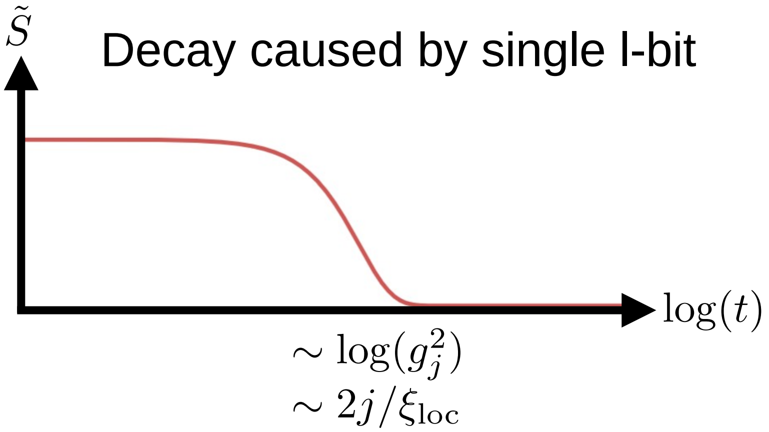

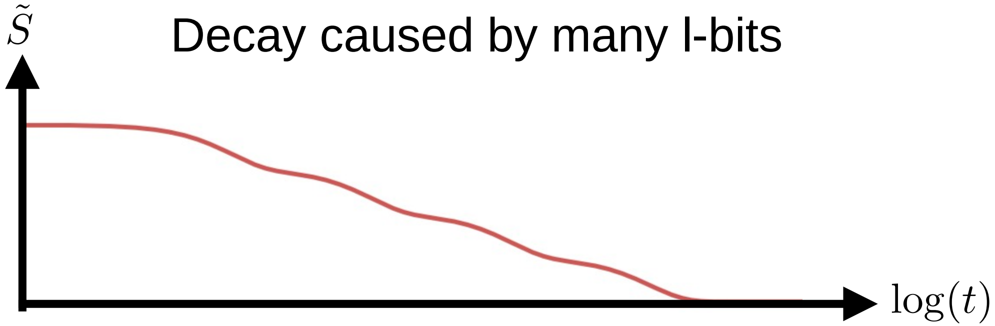

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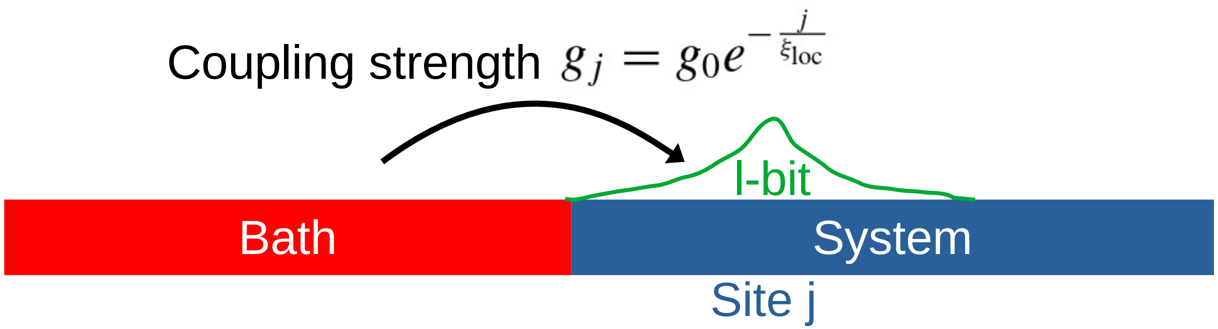
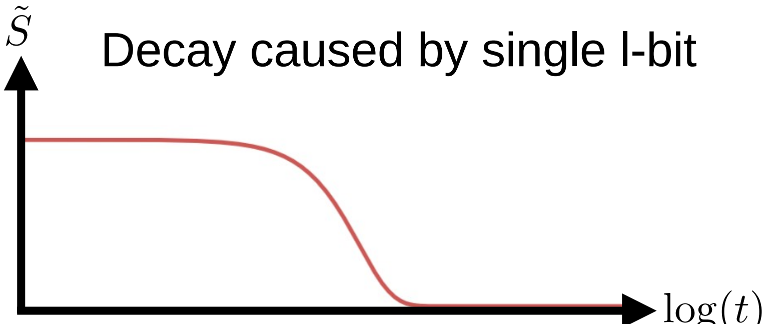
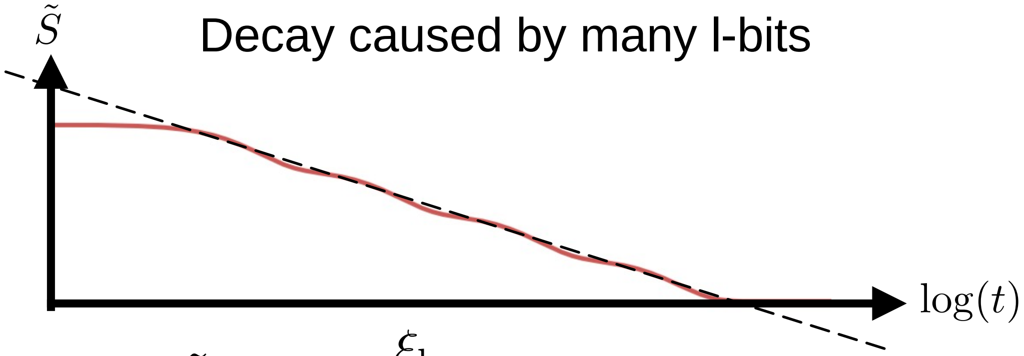


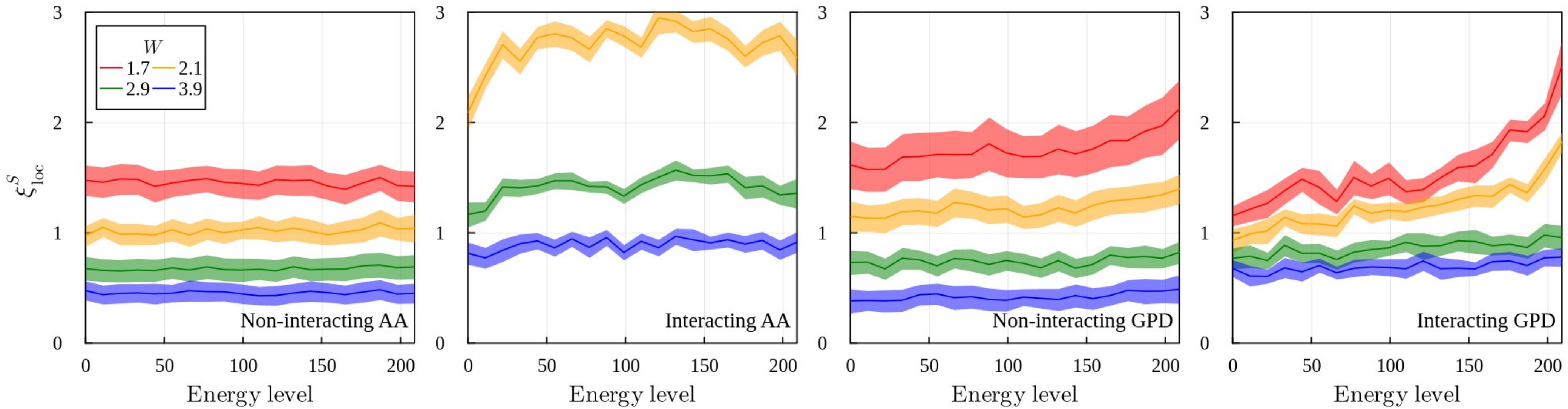


$\sim \log(g_j^2)$
$\sim 2j/\xi_{\mathrm{loc}}$



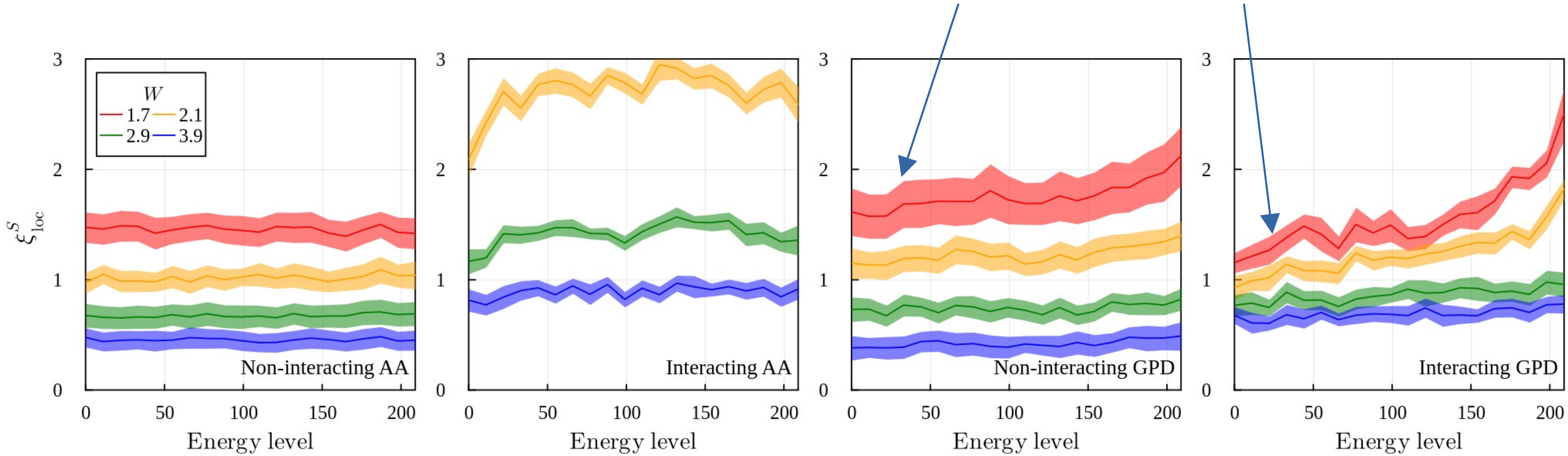
$$\tilde{S}(t) \approx -\frac{\xi_{\mathrm{loc}}}{2L_s} \log t + \mathrm{const}$$

Results – Energy-resolved localization lengths



Results – Energy-resolved localization lengths

Interaction can strengthen localization!



Conclusion

- We couple a bath to a quasiperiodic spin chain
- From the growth of entanglement entropy, we can distinguish the ETH, non-ergodic extended, and localized regimes
- We can extract the localization length in the localized regime, showing that interaction can strengthen localization in the GPD model.