Anomalous Time Crystals

Yi-Ting Tu, David M. Long, Dominic Else

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arXiv: in preparation

Floquet time crystal

Decomposition of Floquet cycle:

$$U_F = e^{-iH_F T} X$$

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$$H_F = \sum_i J_i \sigma_i^z \sigma_{i+1}^z + \cdots , \quad X = \prod_i \sigma_i^x$$

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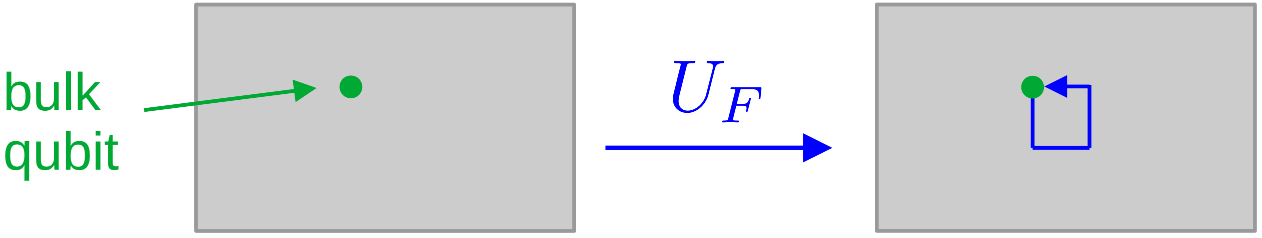

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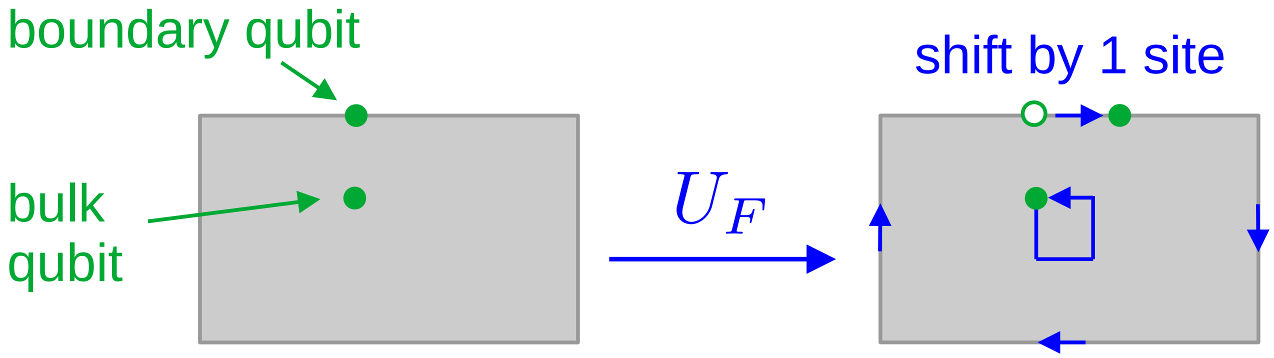

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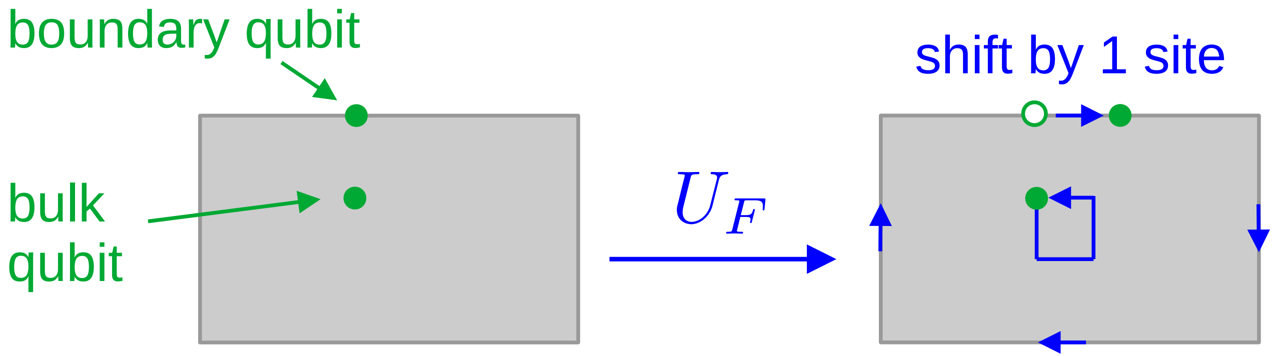
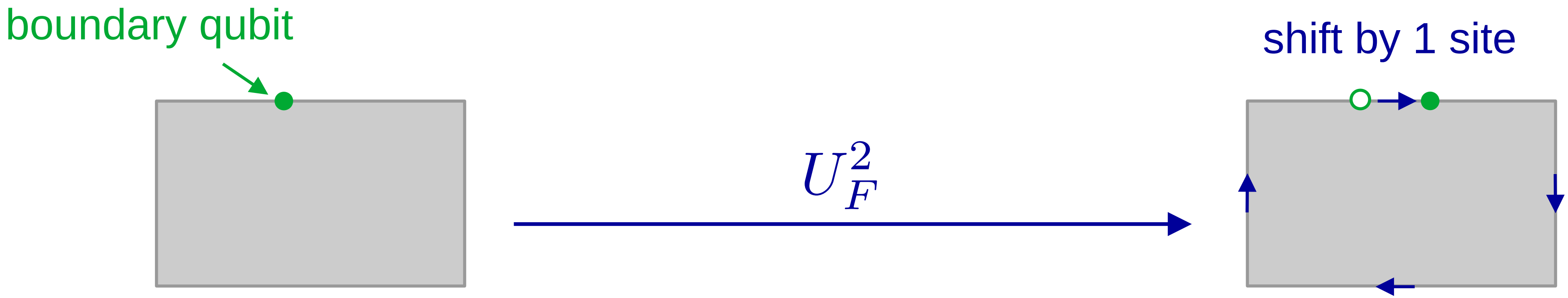


"Fractional" (radical) chiral Floquet phase: [Po2017]



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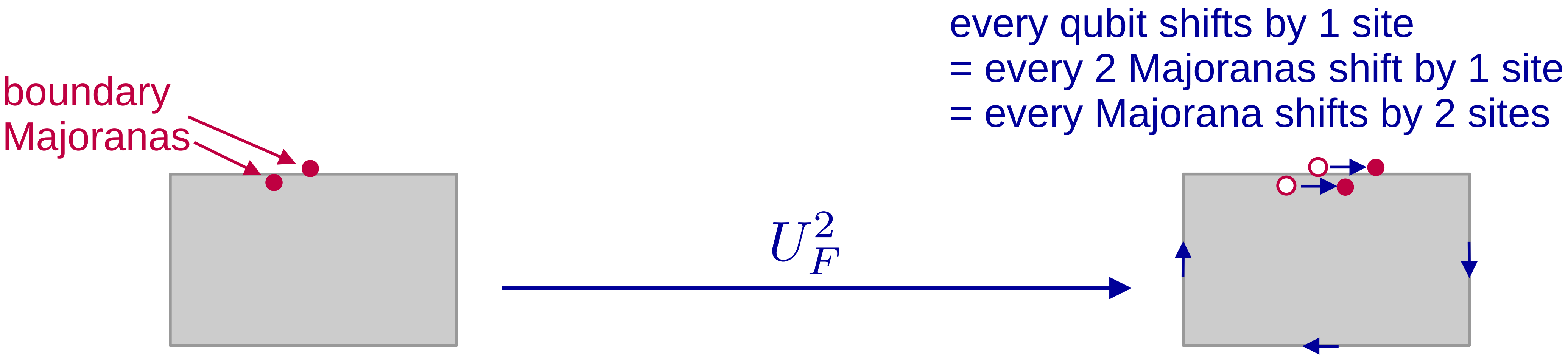
How to shift "half a boundary qubit" per cycle on a bosonic lattice?



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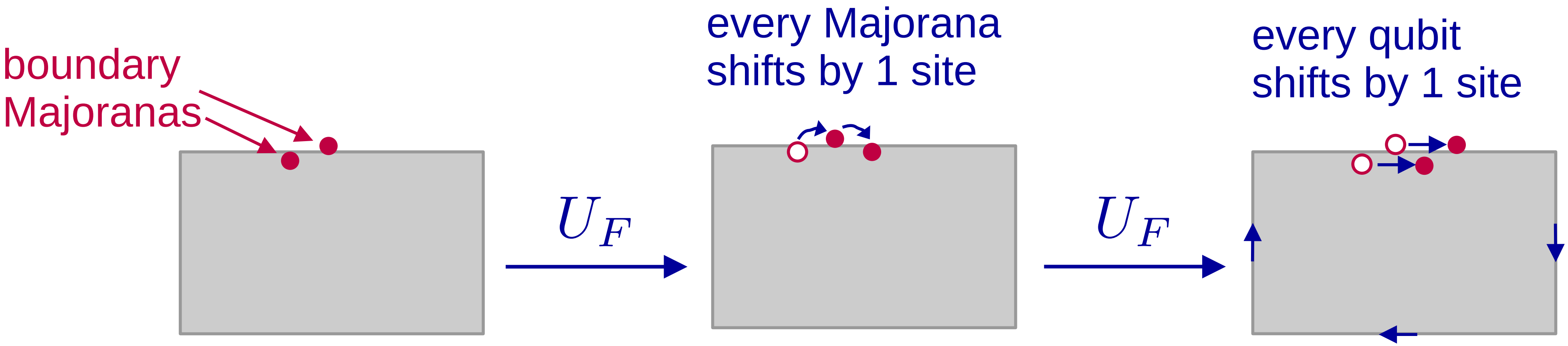

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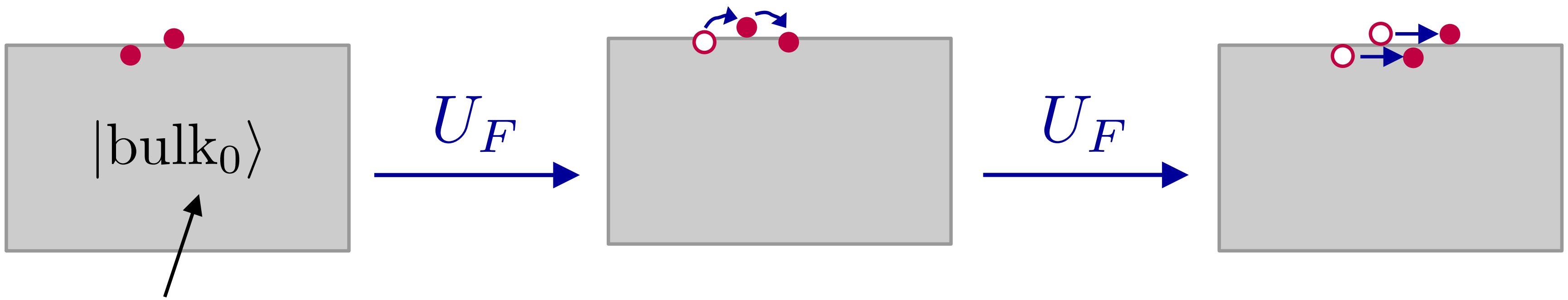

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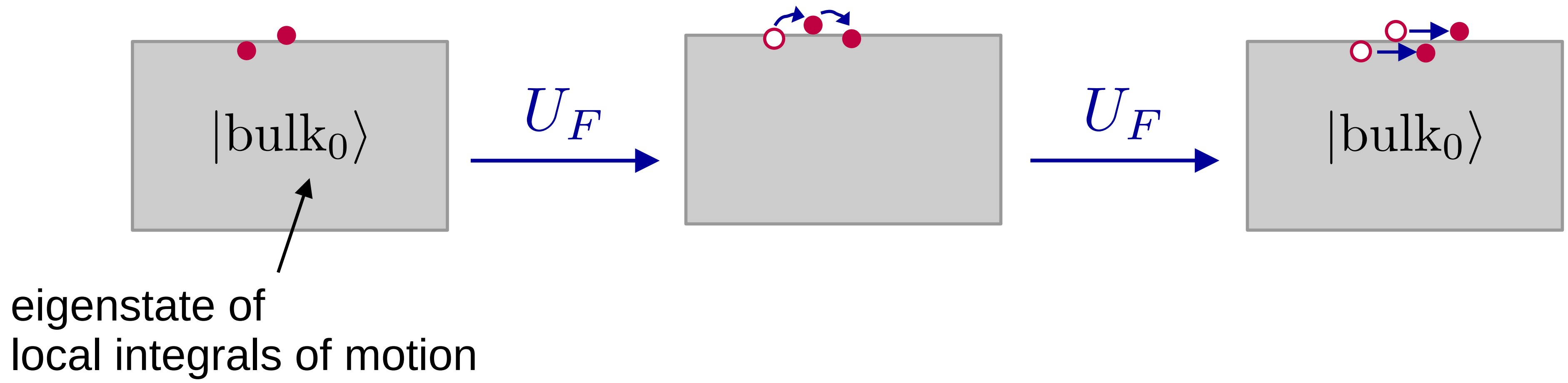

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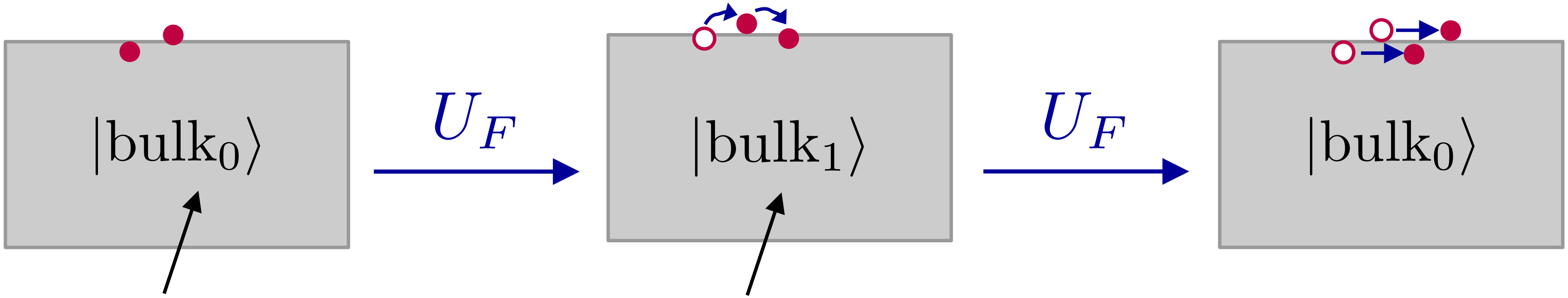
Single Majoranas are not "physical"
and does not exist without the bulk!



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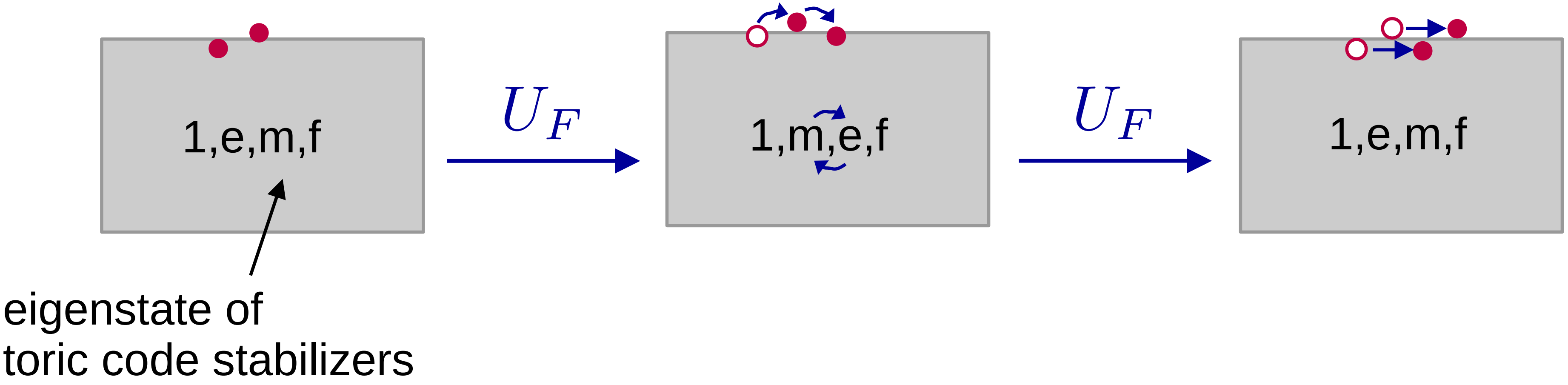

Radical CF phase as time crystal

$$U_F = e^{-iH_F T} X$$

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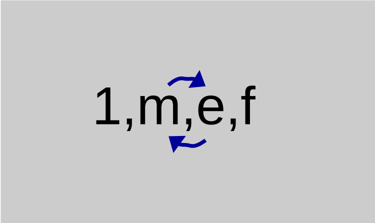
$X =$ 

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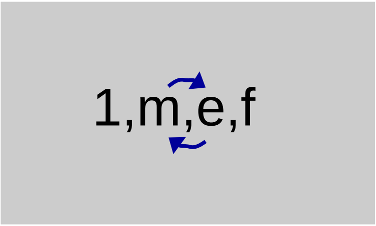
$X =$ 

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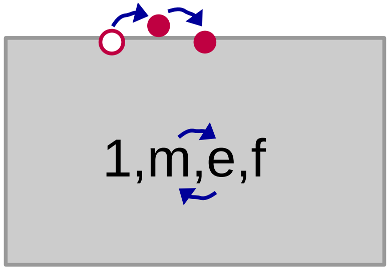
$X_{\text{obc}} =$ 

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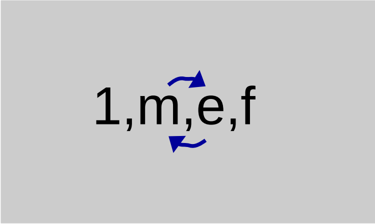
$X =$ 

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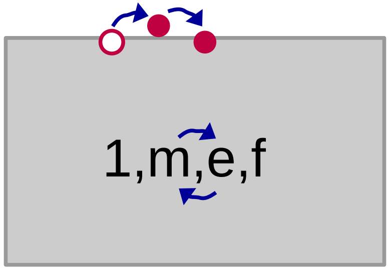
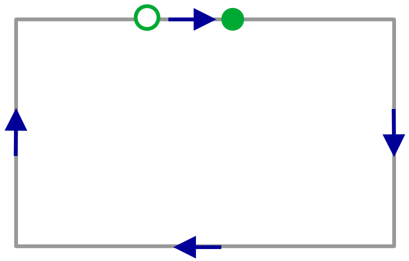
$X_{\text{obc}} =$ 

$X^2_{\text{obc}} =$  $\neq 1$

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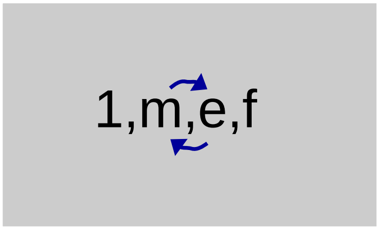
$X =$ 

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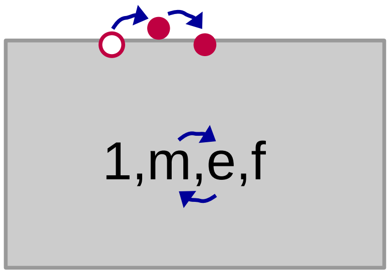
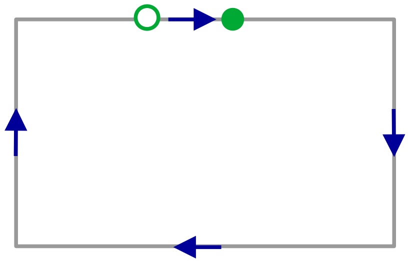
$X_{\mathrm{obc}} =$ 

$X^2_{\mathrm{obc}} =$  $\neq 1$

"anomalous"

Anomaly of a lattice symmetry

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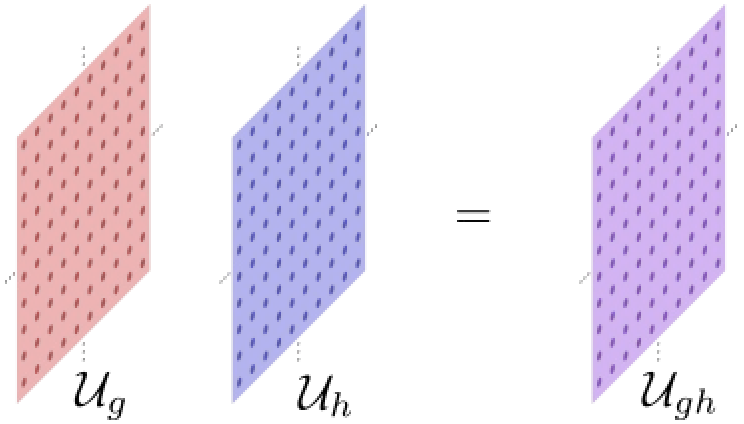
$$\mathcal{U} : G \to \mathrm{QCA}_d$$

Quantum cellular automata
(locality-preserving unitaries)

$d{=}2$ here



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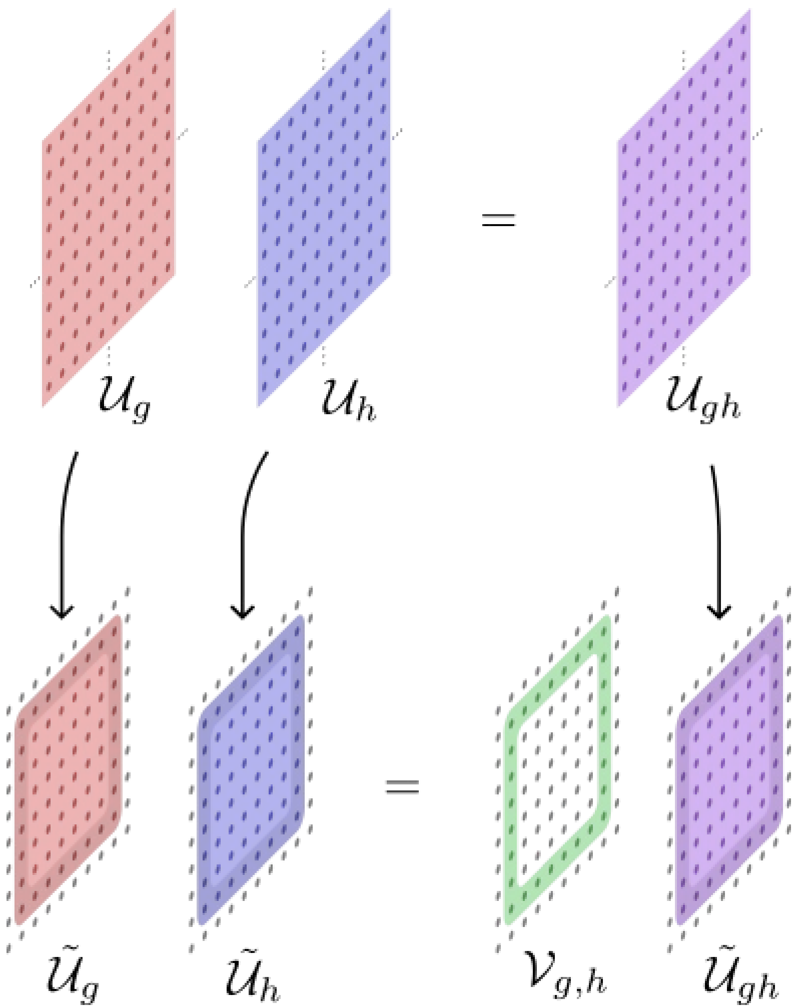

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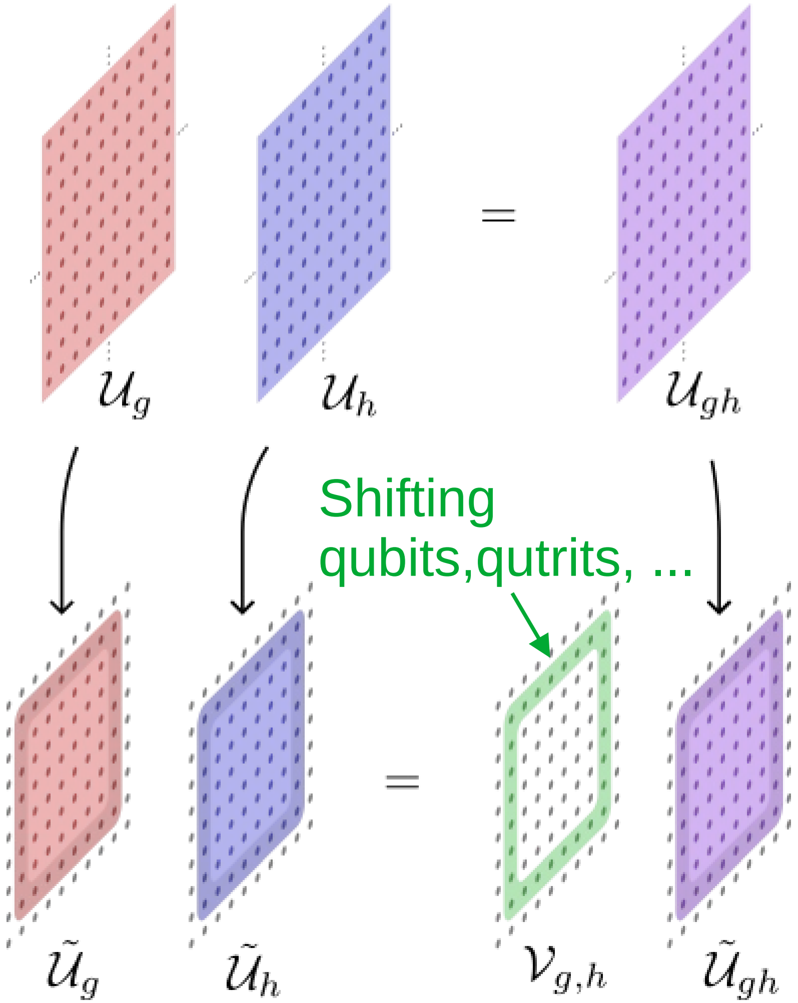

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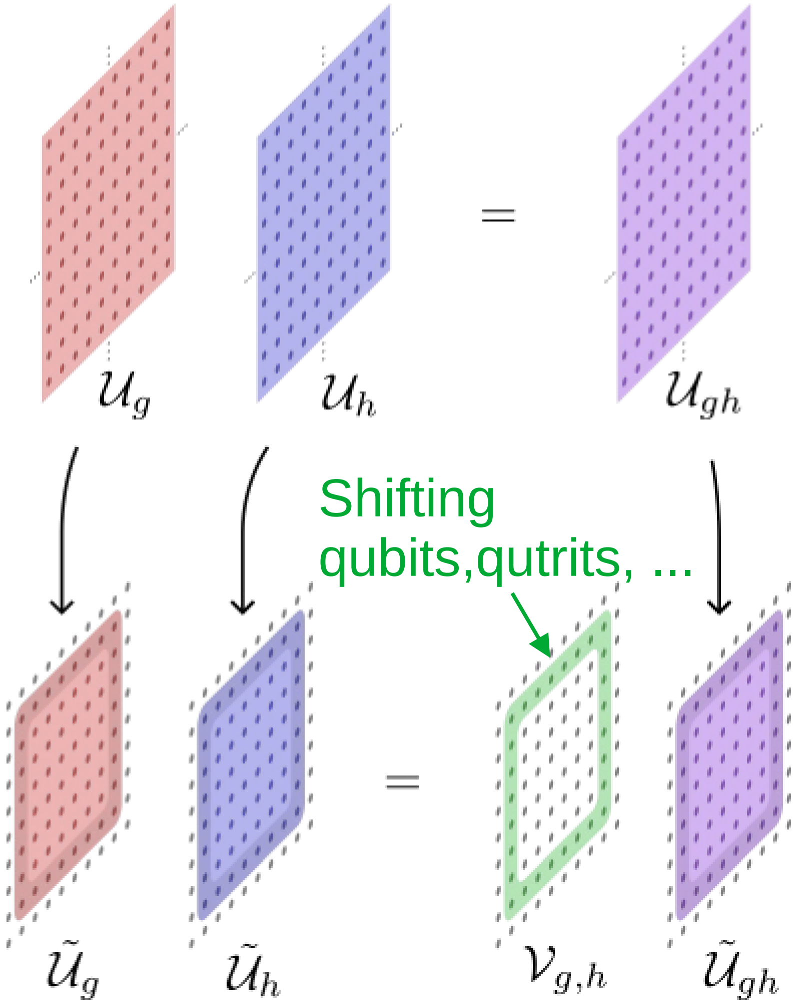

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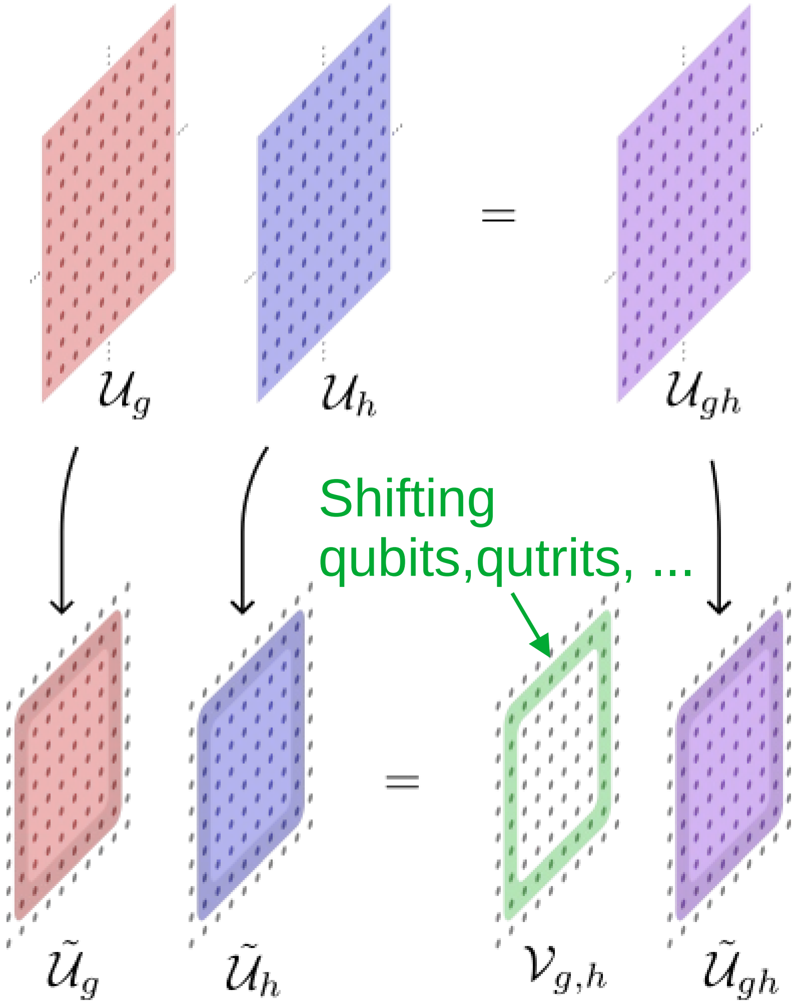

Conclusions

- *Anomalous time crystals* fail to be time crystals (of the same N) on OBC.
- The *radical chiral Floquet phase* is an example.
- They are classified by the anomaly of the $\mathbb{Z}_N$-rep X characterized by the group cohomology $H^2[\mathbb{Z}_N, \pi_0(\mathrm{QCA}_{d-1})]$