Avalanche stability transition in interacting quasiperiodic systems

Yi-Ting Tu

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APS March Meeting 2023

Motivation

- Both of these disorders lead to finite size MBL:
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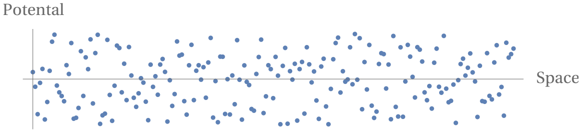
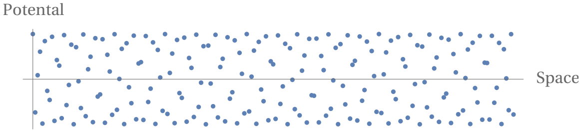


  - Quasiperiodic



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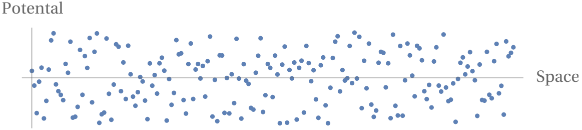

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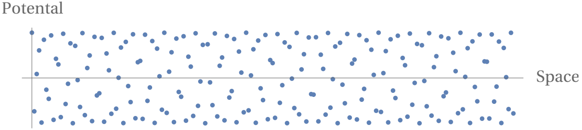

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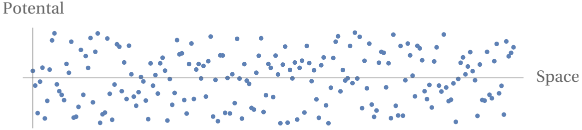

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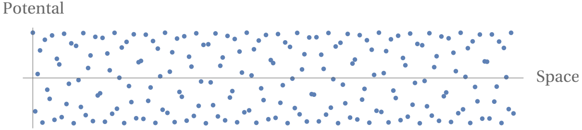

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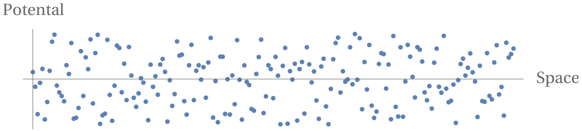

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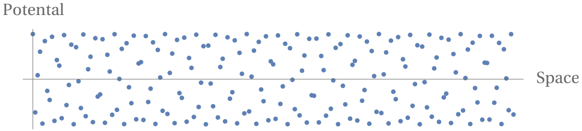

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- **Is quasiperiodic MBL more “stable” than random?**

The models

Spin-$\frac{1}{2}$ Heisenberg model with on-site disorder:

$$H = \frac{1}{4}\sum_{j=1}^{L-1} \vec{\sigma}_j \cdot \vec{\sigma}_{j+1} + \frac{1}{2}\sum_{j=1}^{L} h_j Z_j, \tag{1}$$

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- W : disorder strength

Outline

Background: Avalanche instability

Open system simulation approach

The real space RG approach

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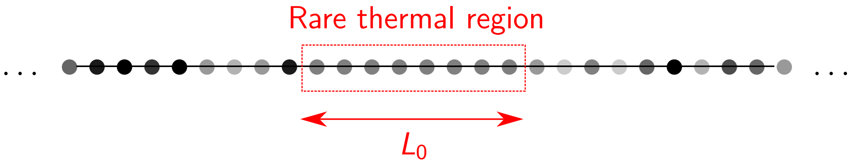

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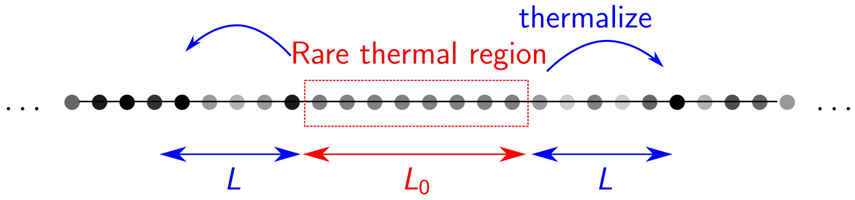

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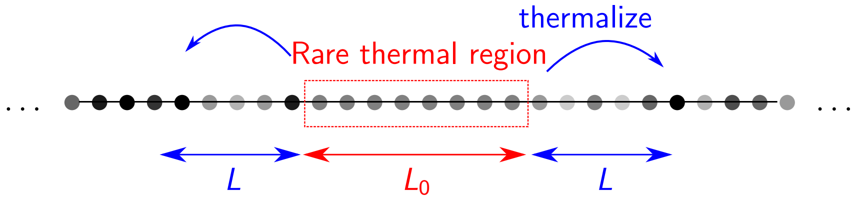

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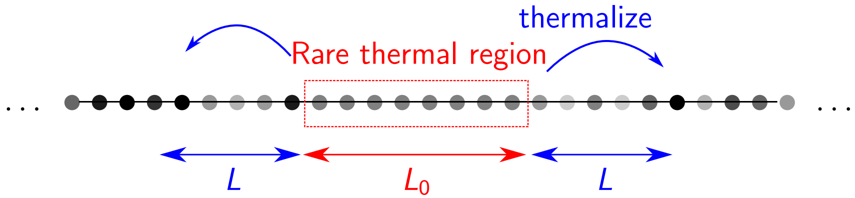

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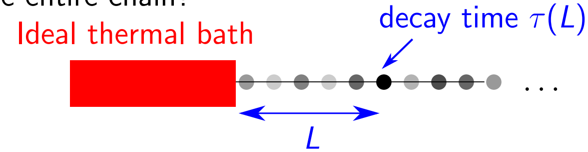

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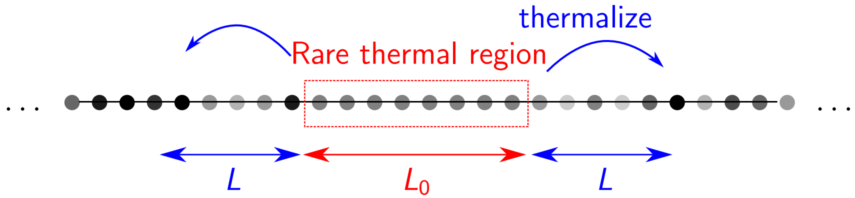

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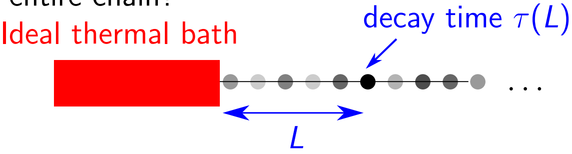

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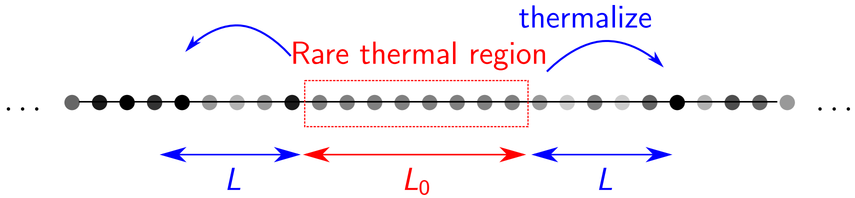

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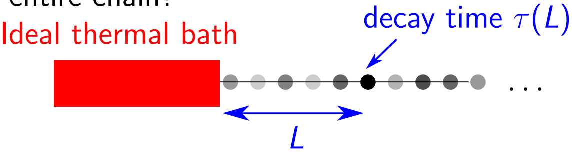

- Compare: Ideal decay rate $1/\tau(L)$ v.s. Non-ideal level spacing $1/(2^{L_0+2L})$.
- $\tau(L)$ grows faster than $4^L \implies$ thermodynamic MBL
- $\tau(L)$ grows slower than $4^L \implies$ entire system will thermalize

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Coupling to the bath [Morningstar et al. '21]

Consider an open chain of length L.

$$\mathcal{L}[\rho] = -i[H,\rho] + \gamma \sum_{\mu} \left(L_\mu \rho L_\mu^\dagger - \frac{1}{2}\{L_\mu^\dagger L_\mu, \rho\} \right), \tag{4}$$

where

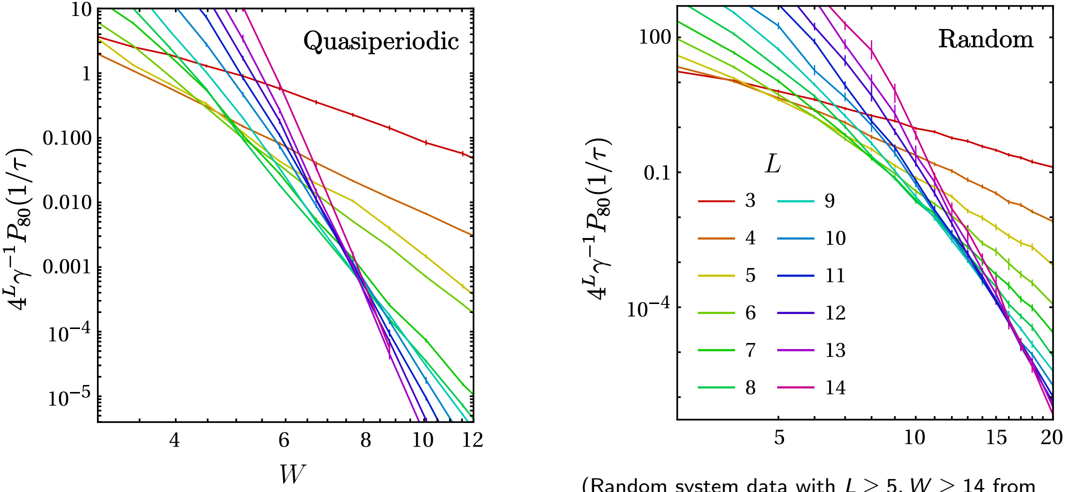
- $L_\mu = (X_0, Y_0, Z_0)$,
- γ: Bath coupling strength

First-order perturbation approach: $\gamma \ll 1$

$$\tau(L) \approx \frac{1}{\lambda_1}, \quad \lambda_1 : \text{ largest negative eigenvalue of } \mathcal{L}_{nm} = \langle m|\mathcal{L}[|n\rangle\langle n|]|m\rangle \tag{5}$$

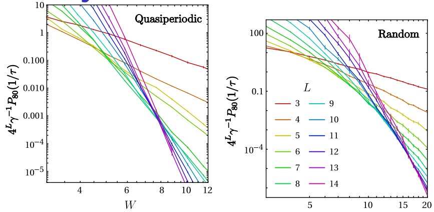
where $|n\rangle$'s are the eigenstates of H.

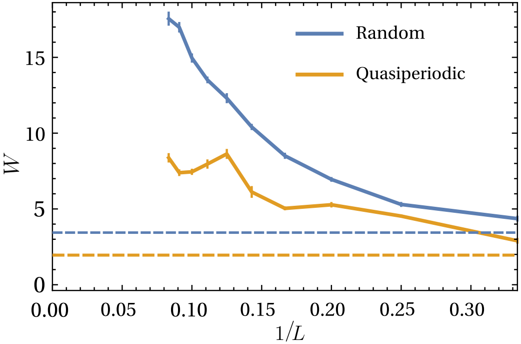
Open system simulation results



(Random system data with $L \geq 5$, $W \geq 14$ from Morningstar et al. '21)

Open system simulation results (intersections)





Solid: Intersection of L and $L+2$ in the figures above.
Dashed: Finite size W_{critical} from level statistics ($L \sim 16$).

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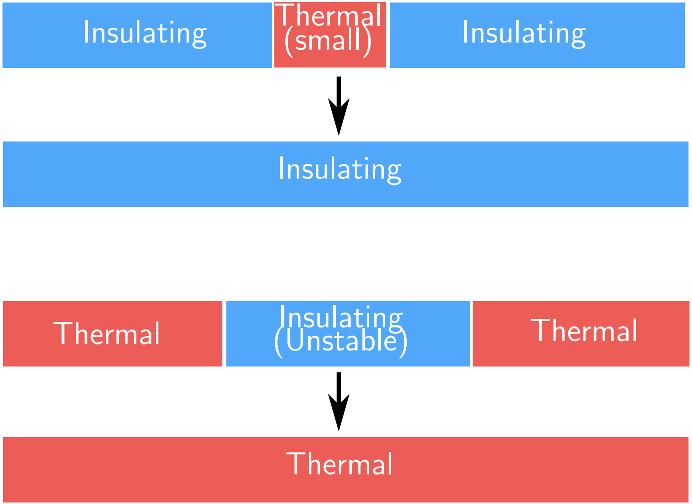
Idea of the real space RG approach [Morningstar, Huse '19]



The RG rules [Morningstar, Huse '19]

Parameters of the blocks:

- Thermal block: (Physical length L)
- Insulating block: (Physical length L, Primary length d)
  $d = \infty$: perfect insulation
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- Begin with a sequence of blocks: thermal, insulating, thermal, insulating, ...

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- When $\Lambda = d_n$ of an insulating block, perform the decimation:
  $(L_{n-1}), (L_n, d_n), (L_{n+1}) \to (L_{n-1} + L_n + L_{n+1})$
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- The type of the final block determines the phase of the system.

The initial blocks

- Define the quantity for each link:

$$\xi_{j,j+1} = \left(\frac{1}{\Delta h_j^2} \frac{1}{|\Delta h_j + 1|} \frac{1}{|\Delta h_j - 1|} \right)^{\alpha/4}, \quad \Delta h_j = h_{j+1} - h_j \tag{6}$$

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- n consecutive $\xi < 1$ links $\rightarrow$ 1 insulating block $(L = n, d = \sum(\frac{1}{\xi} - 1))$
- $\alpha = 0.1$ is chosen to fit the critical W's from the open-system simulation (both random and quasiperiodic at the same time)

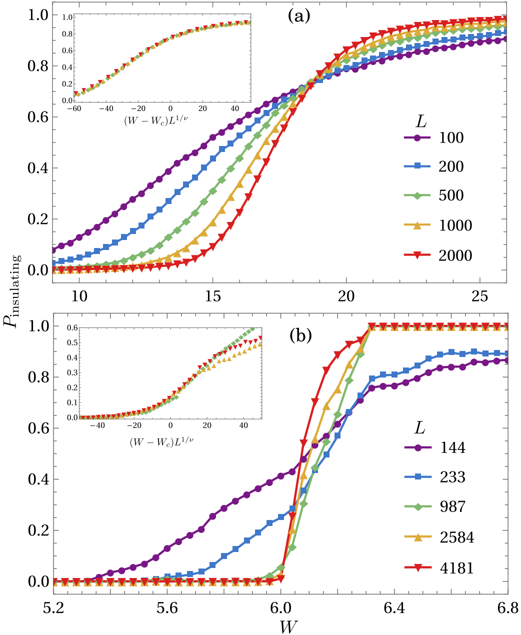
RG results

(a) Random

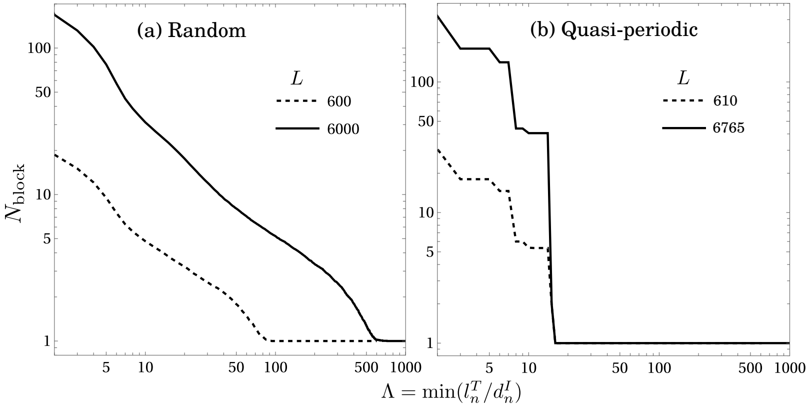
- $W_c = 18.9$
- slow convergence
- $\nu = 2.8$
  $\approx$ random blocks
  [Morningstar, Huse '19]

(b) Quasiperiodic

- $W_c = 6.0$
- fast convergence
- $\nu = 1.2$
  $\approx$ quasiperiodic blocks
  [Agrawal et al. '20]



Numbers of blocks left v.s. Cutoff length



In the quasiperiodic system, the blocks are decimated abruptly independent of L.

- No asymptotically large thermal seed exists.
- Finite size effects are less severe.

Conclusion

- With avalanche instability study, thermodynamic MBL likely exists for quasiperiodic systems with a moderate disorder, unlike for random systems.
- MBL experiments in many-atom (> 100) 1D systems may benefit from quasiperiodic potential.

level statistics approach (mean level spacing ratio)

$$\langle r \rangle = \left\langle \frac{\min\{\delta_n, \delta_{n-1}\}}{\max\{\delta_n, \delta_{n-1}\}} \right\rangle, \quad \delta_n = E_{n+1} - E_n \tag{7}$$

(Periodic b.c. and only consider zero-magnetization sector)

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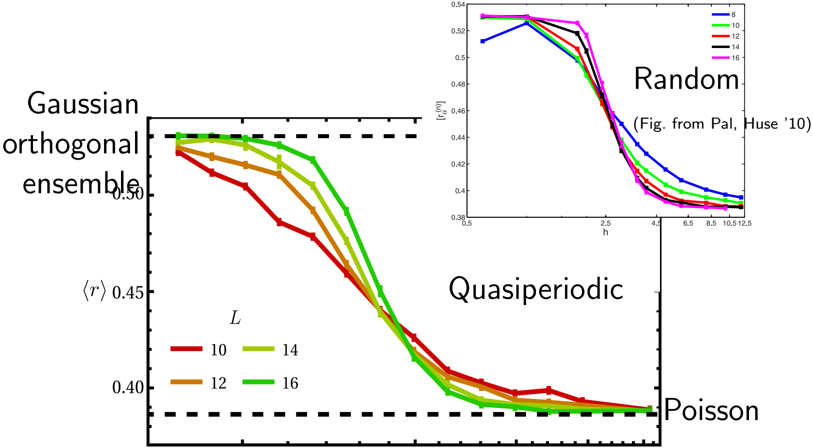

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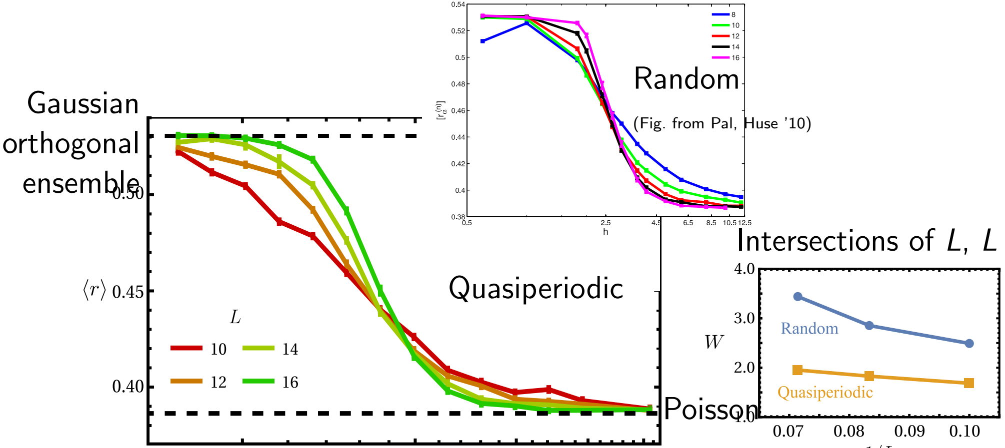
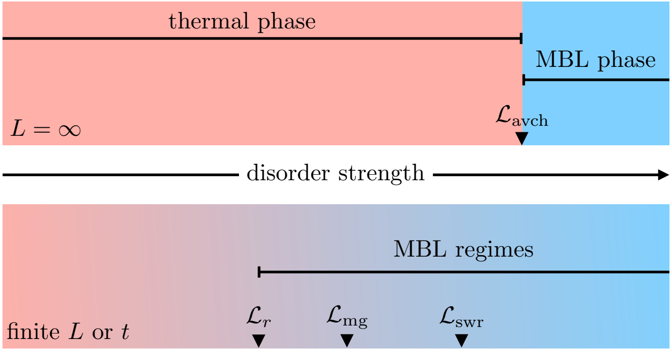


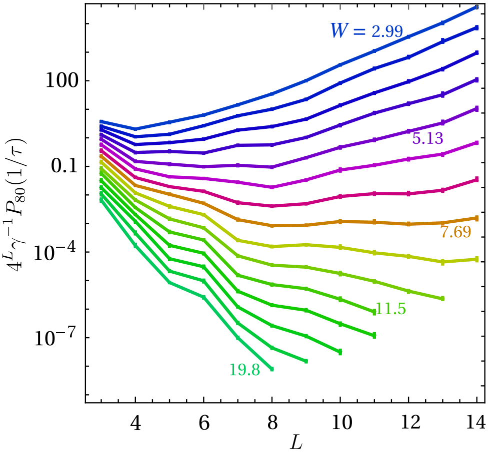
Finite size vs Thermodynamic MBL



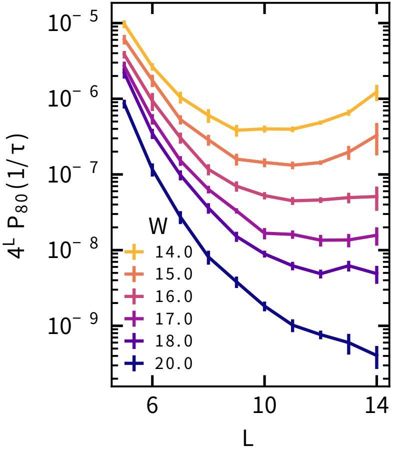
(Fig. from Morningstar et al. '21)

Results: Scaled decay rate vs L

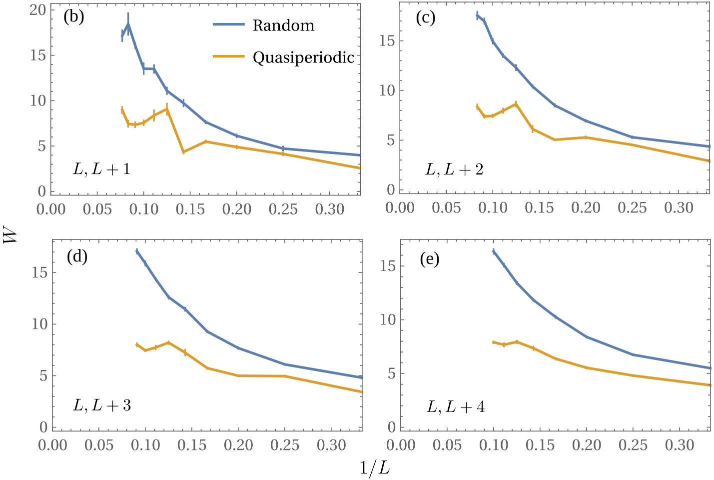
Quasiperiodic



Random (Fig. from Morningstar et al. '21)



Results: The W at the intersections of curves



Results: Other quasiperiods