Many-body localization in Quasiperiodic systems

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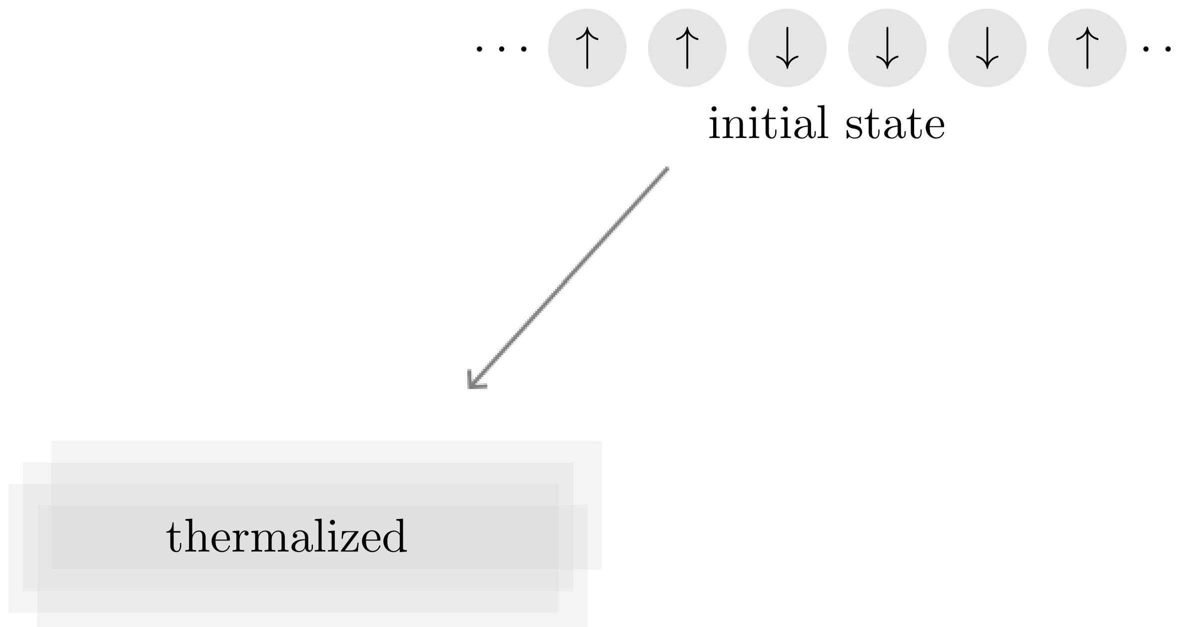
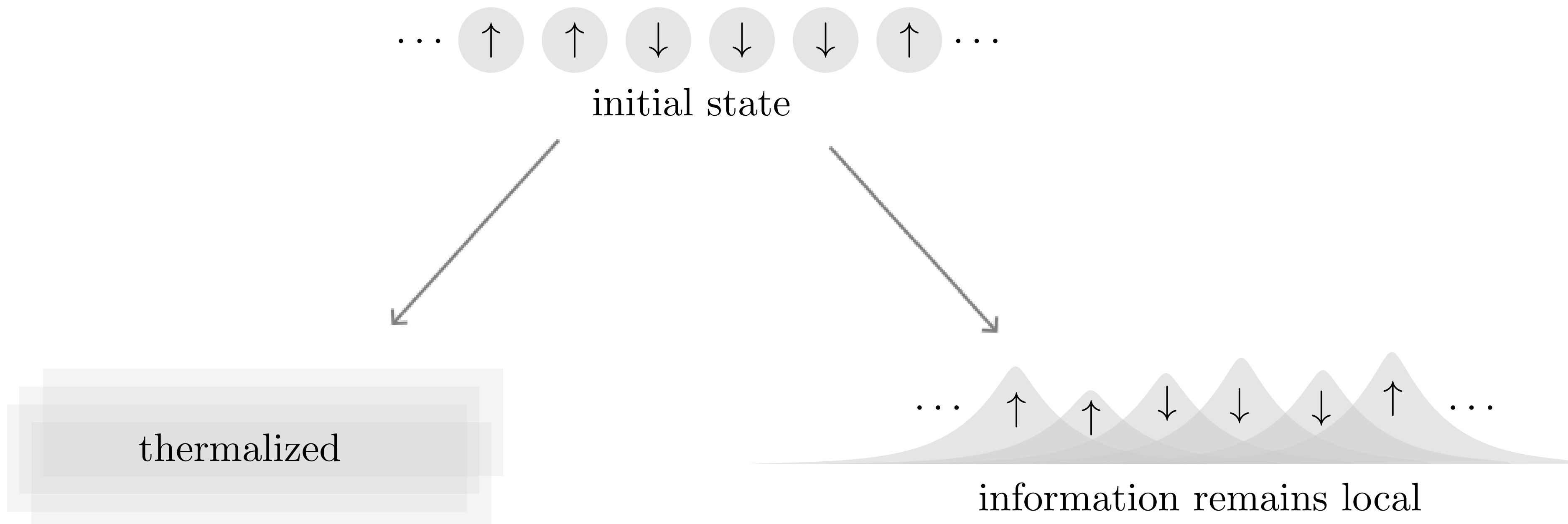
Outline

- Introduction
  - Many-body localization (MBL)
  - Random v.s. Quasiperiodic systems
- Non-ergodic extended (NEE) regime
  - Numerical results
  - Theoretical explanation
- Many-body critical (MBC) phases
  - Single-particle hierarchical mirror structure (HMS)
  - Effects of interaction on HMS
  - Asymptotically solvable model

$$\cdots \uparrow \uparrow \downarrow \downarrow \downarrow \uparrow \cdots$$

initial state

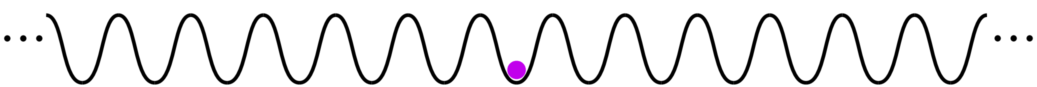

$\cdots \uparrow \uparrow \downarrow \downarrow \downarrow \uparrow \cdots$
initial state
thermalized



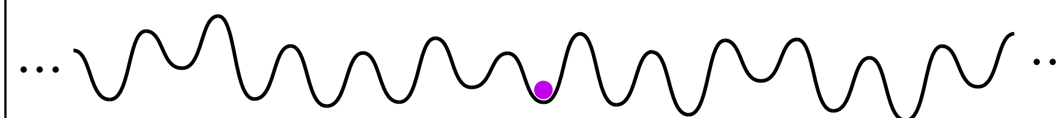
... ↑ ↑ ↓ ↓ ↓ ↑ ...
initial state
thermalized
... ↑ ↑ ↓ ↓ ↓ ↑ ...
information remains local

Single particle extended/localized systems

A local particle in a periodic potential:



A local particle in a random potential:



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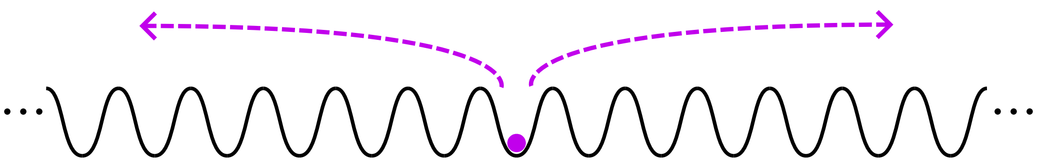
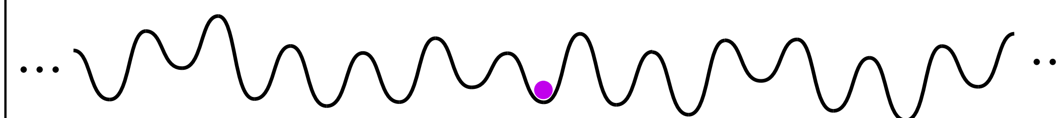


Travels further and further away
and never comes back

A local particle in a random potential:



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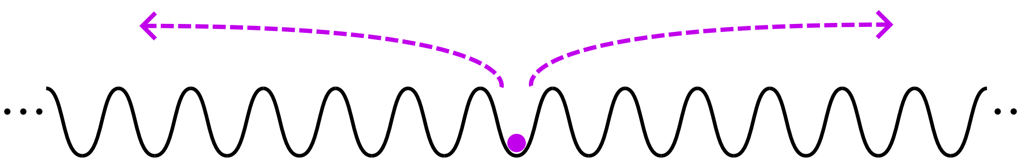

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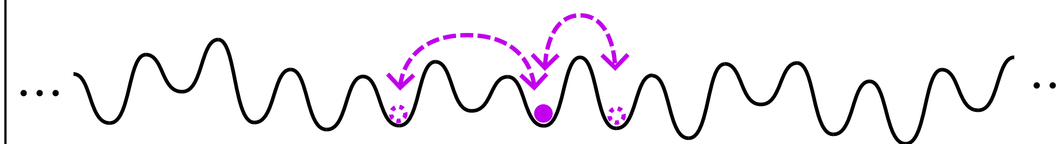
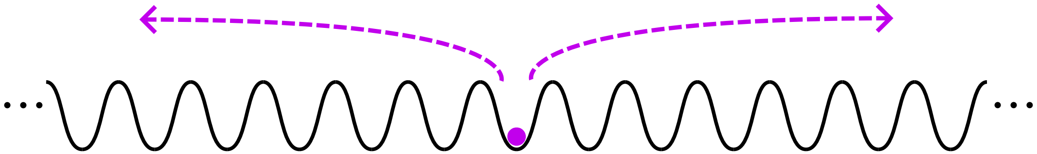


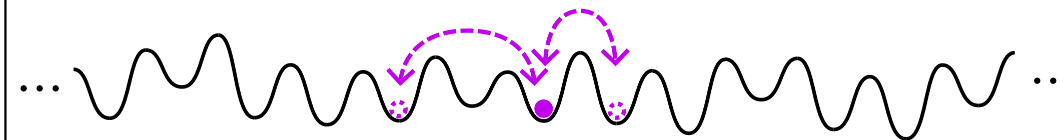
Comes back regularly
and does not go arbitrarily far away
[Anderson1958]

Many-body counterparts of extendedness/localization

Extended particle:



Localized particle:



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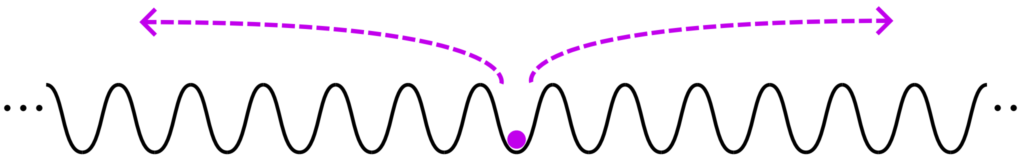

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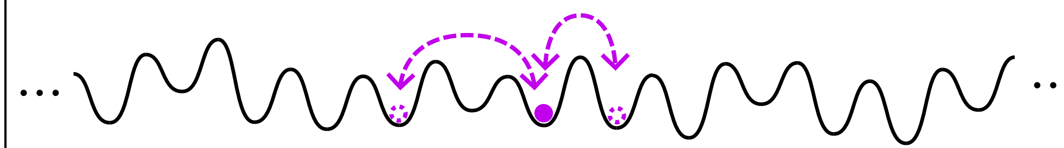

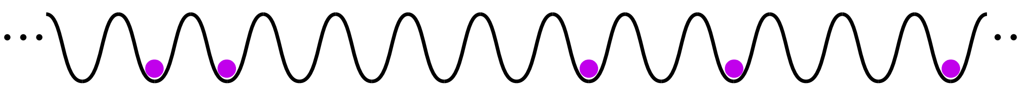

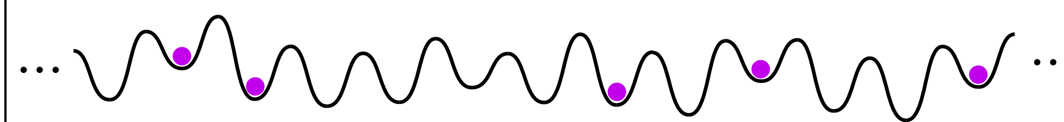

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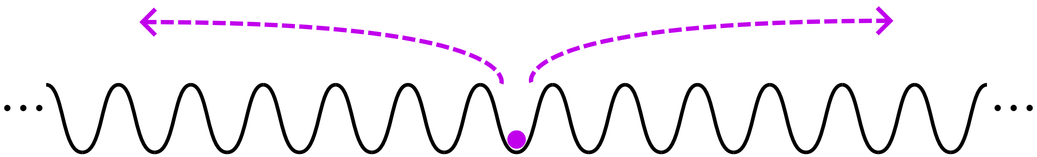

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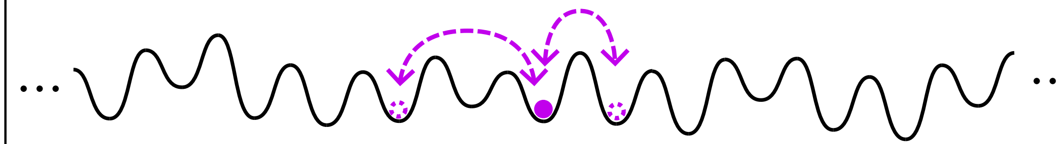

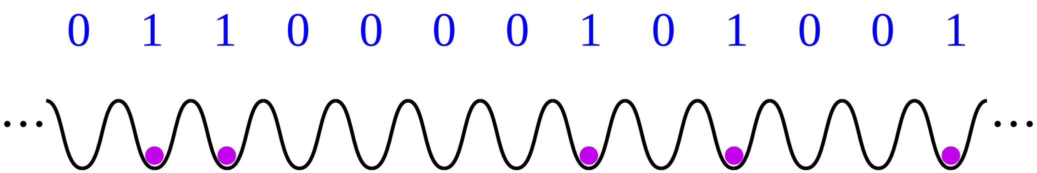

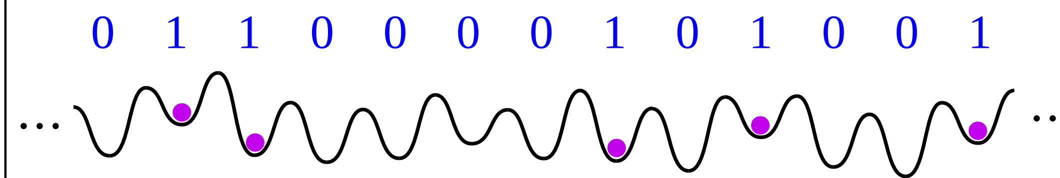

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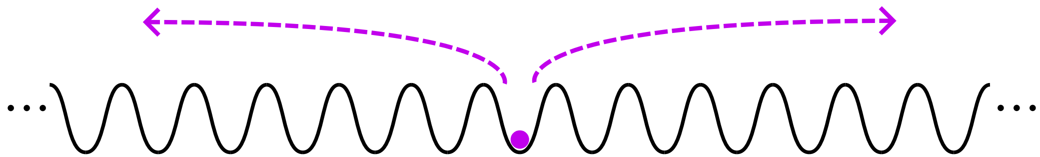

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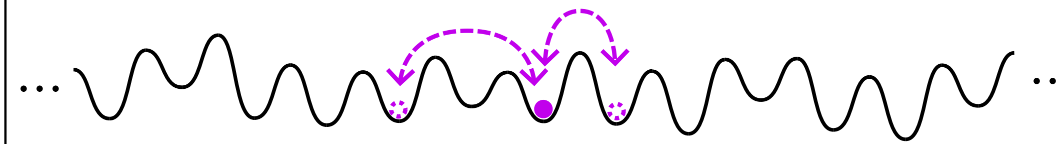
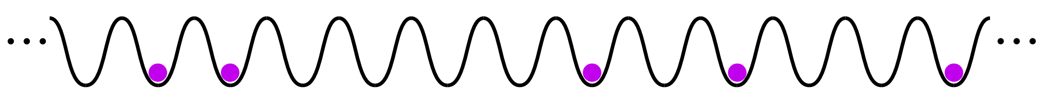


0 1 1 0 0 0 0 1 0 1 0 0 1



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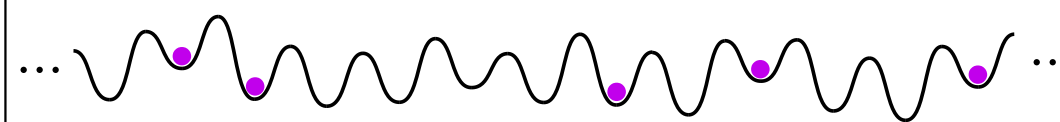

Infinite time

Information spreads out to infinite
Everything looks the same (thermal) locally

Many-body counterparts of extendedness/localization

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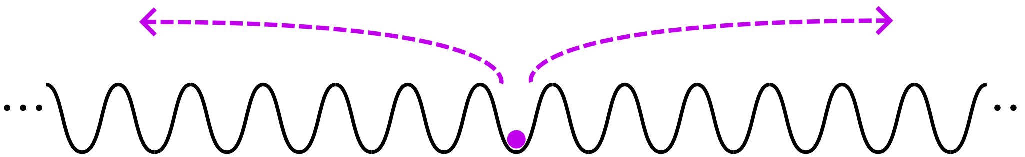

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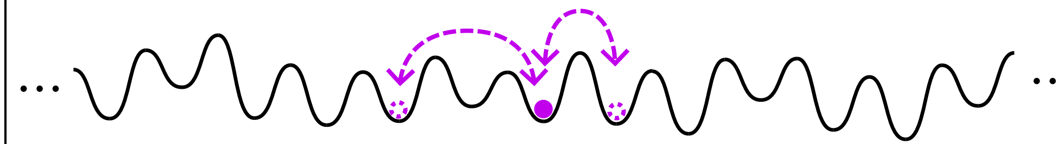

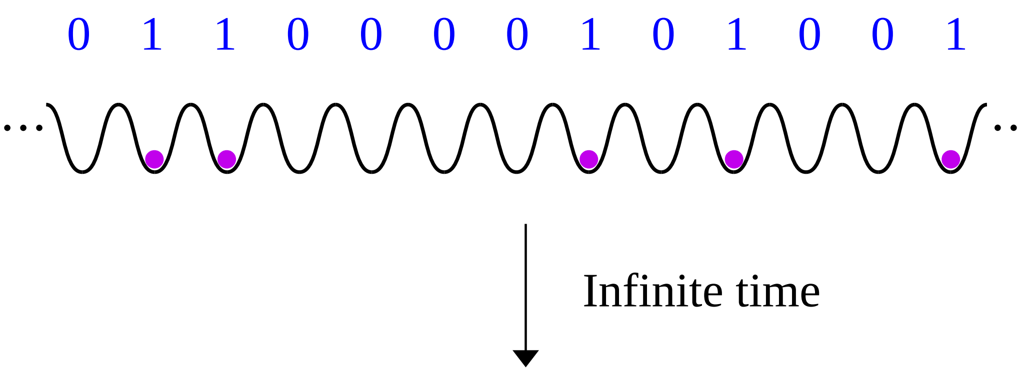

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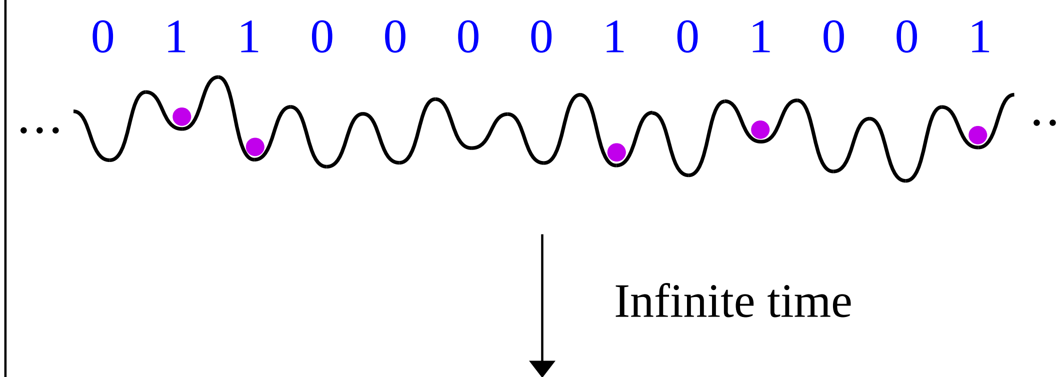

Information only spreads out a little
Memory remains in local observable expectations

Many-body counterparts of extendedness/localization

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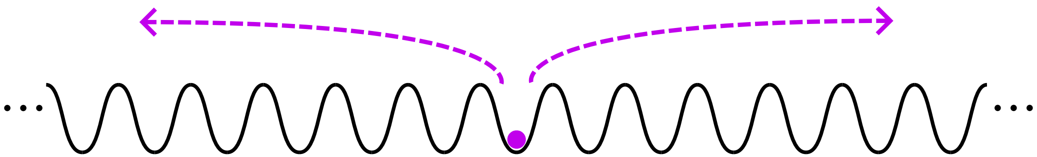

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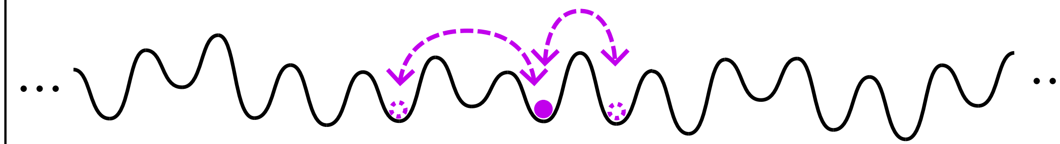

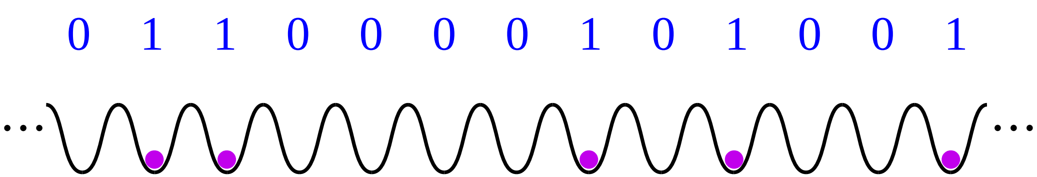

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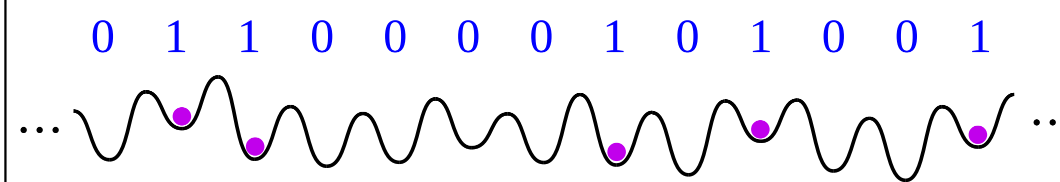
ETH phase



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MBL phase

Many-body counterparts of extendedness/localization

ETH phase	MBL phase

Many-body counterparts of extendedness/localization

ETH phase

Satisfies the eigenstate thermalization hypothesis (ETH):

MBL phase

Many-body counterparts of extendedness/localization

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Satisfies the eigenstate thermalization hypothesis (ETH):

$$\langle E_\alpha | O | E_\alpha \rangle \approx \langle O \rangle_{\text{microcanonical}}$$

O: local observable

$|E_\alpha\rangle$: energy eigenstate with E_α in the microcanonical shell

MBL phase

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Has a complete set of local integrals of motion (LIOMs):

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$$\{L_j\}, [H, L_j] = 0$$

L_j: localized around site j

Energy eigenstate uniquely labeled by eigenvalues of $\{L_j\}$

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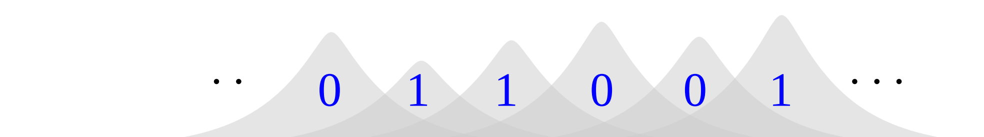

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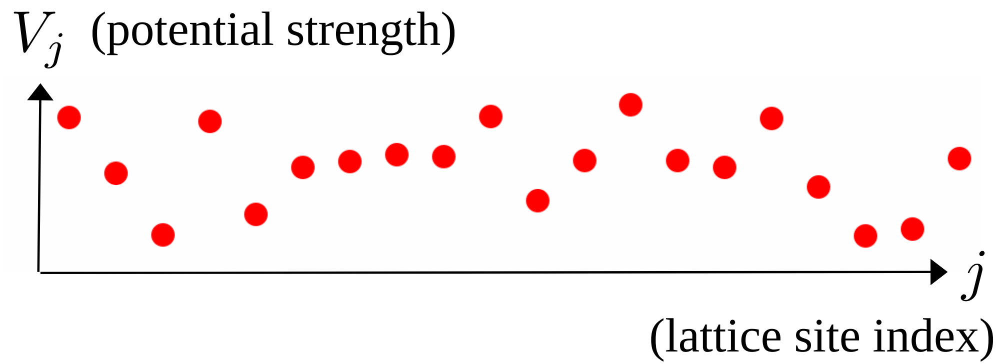
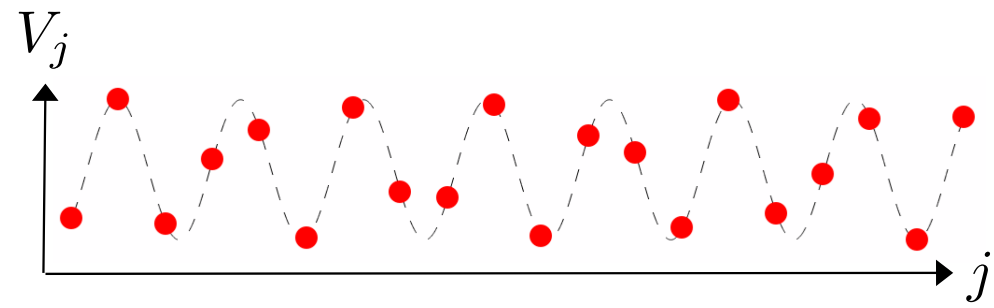
Random systems



Quasiperiodic systems



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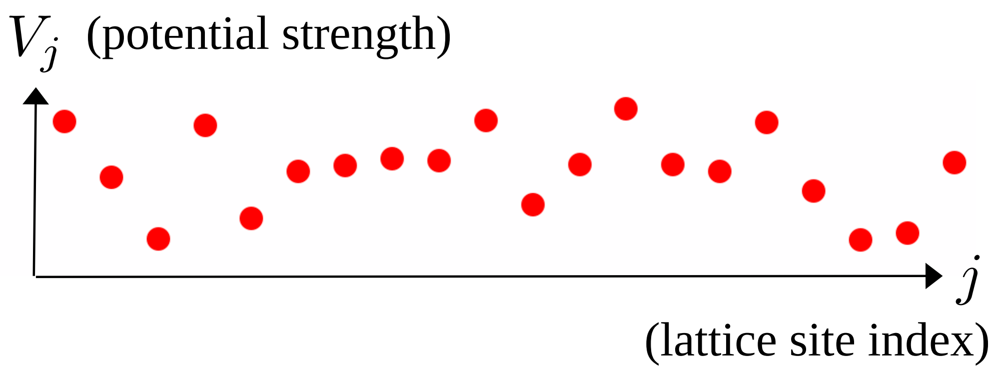
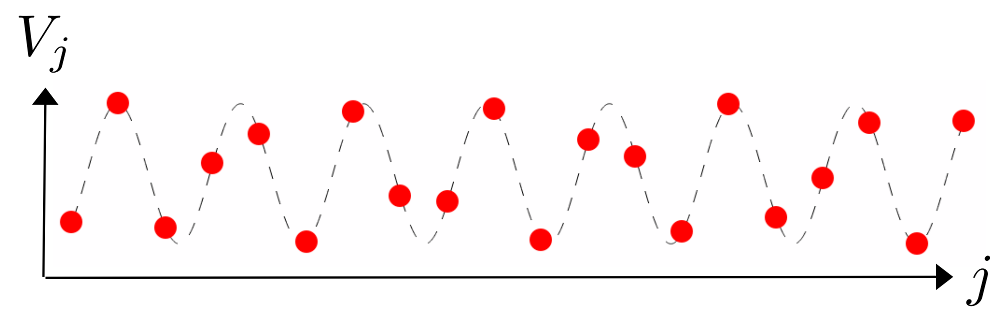


V_j: random numbers indep. for each j

Quasiperiodic systems



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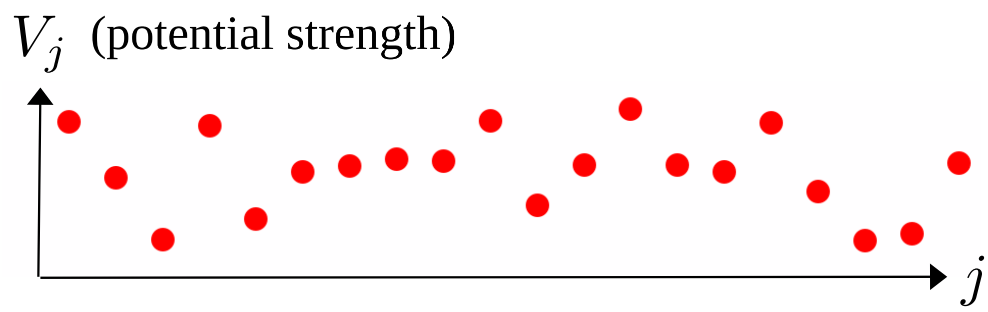

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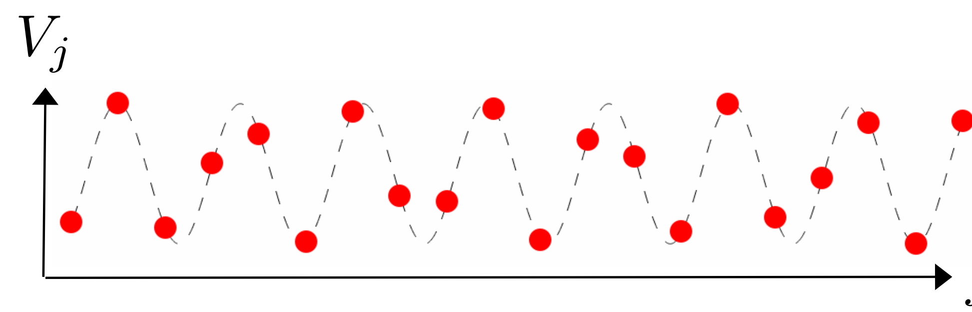
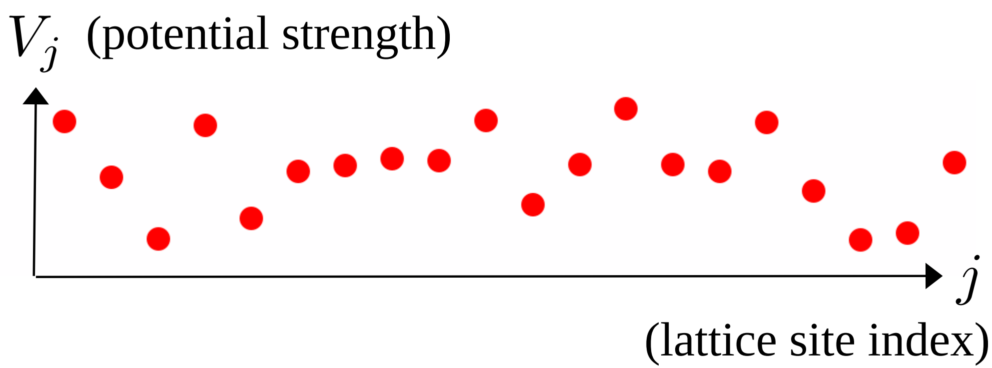


$$V_j = F(\omega j + \phi)$$

$$F(y + 2\pi) = F(y)$$

$$j \in \mathbb{Z}, \qquad \frac{\omega}{2\pi} \notin \mathbb{Q}$$

Random systems



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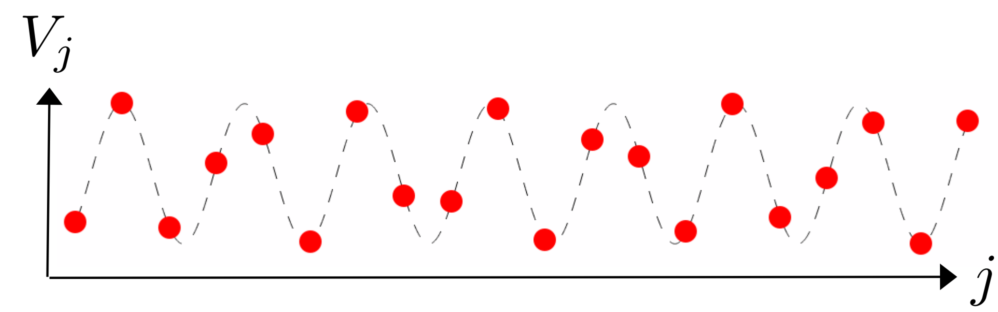

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$$j \in \mathbb{Z}, \qquad \frac{\omega}{2\pi} \notin \mathbb{Q}$$

$$\text{Scale } V_j \in [-W, W], \qquad W:\ \text{disorder strength}$$

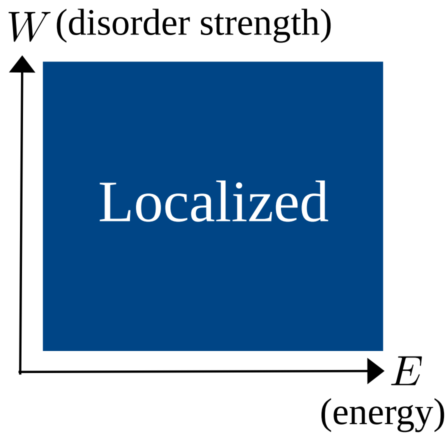
What's special about 1D quasiperiodic systems

	Random (Anderson)	Quasiperiodic
Single particle		
Interacting Many-body		

What's special about 1D quasiperiodic systems

Random (Anderson)

Quasiperiodic



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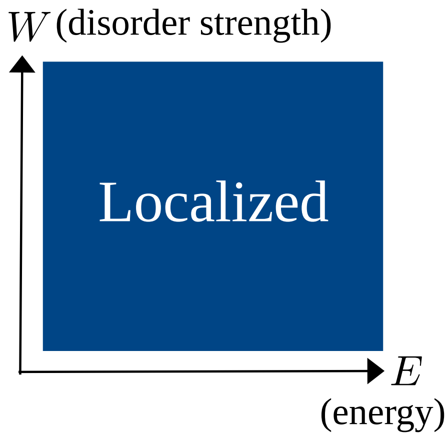
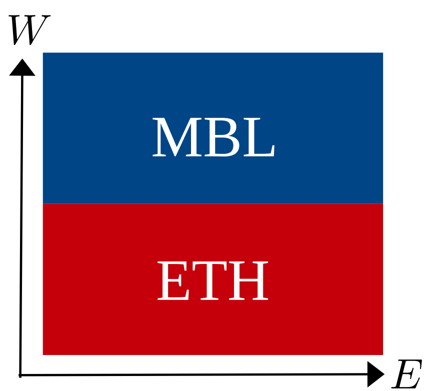


Interacting Many-body

(at least for small size)



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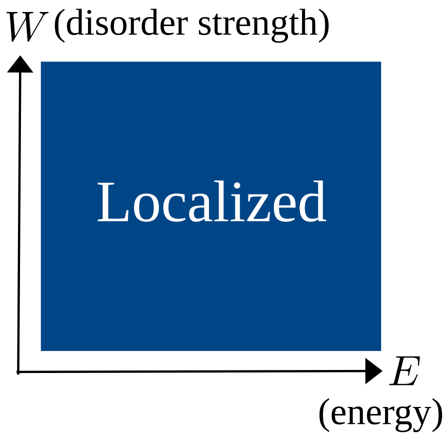
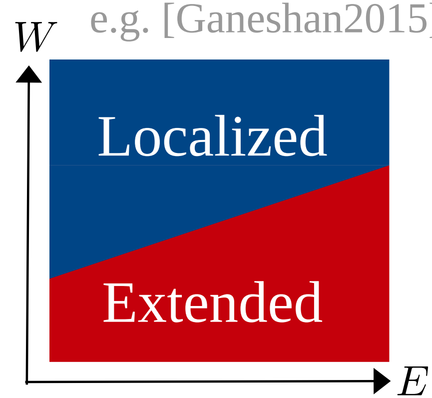
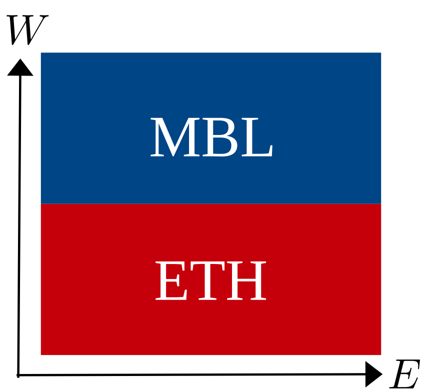
Single particle		e.g. [Ganeshan2015] 
Interacting Many-body (at least for small size)		

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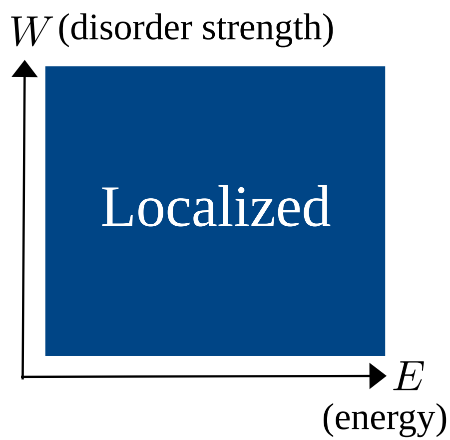

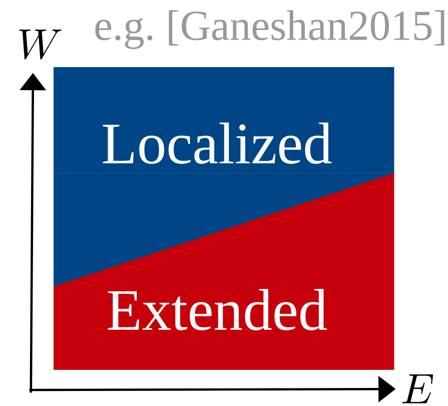

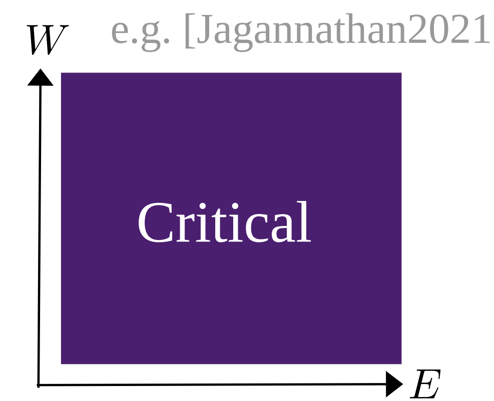

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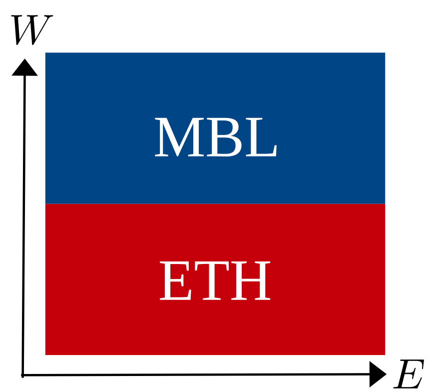

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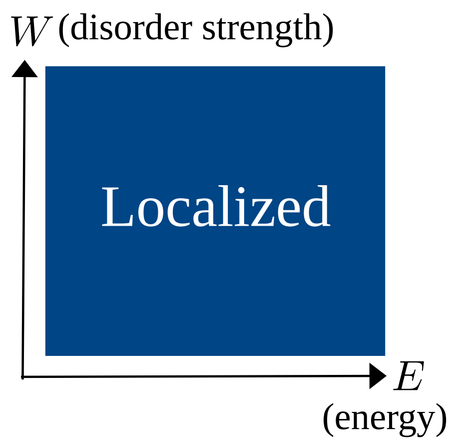
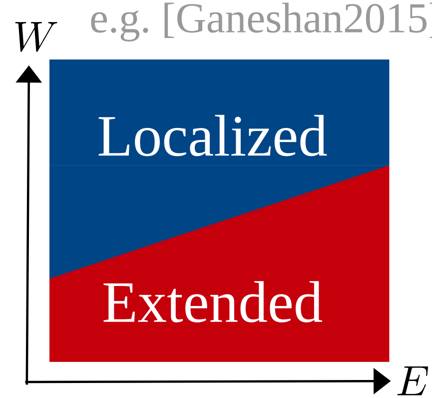
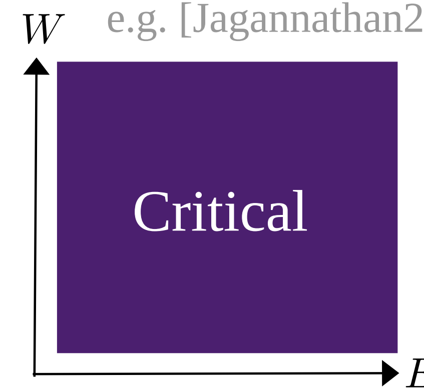


e.g. [Ganeshan2015]



e.g. [Jagannathan2021]

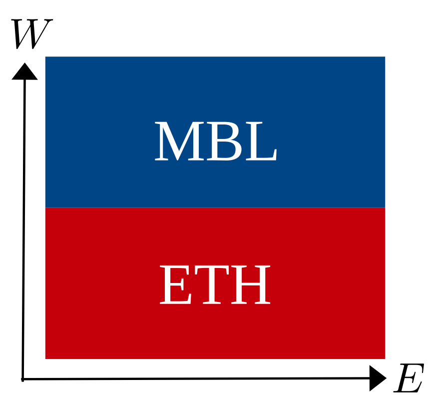


…

(combination of them)

Interacting Many-body

(at least for small size)



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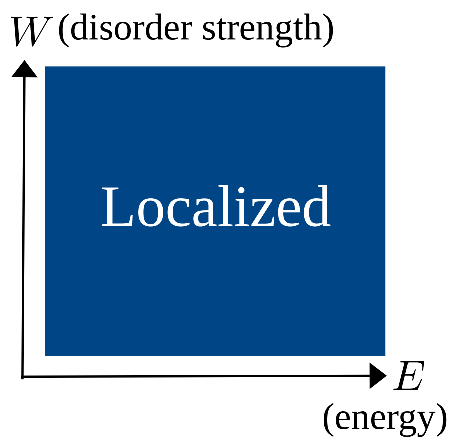

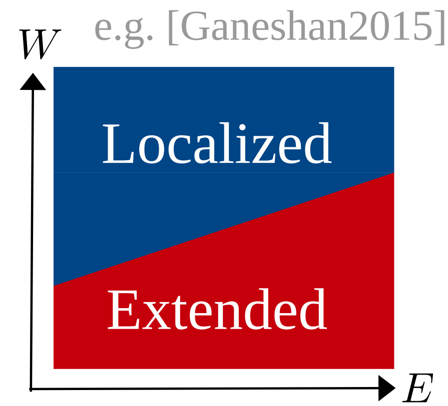

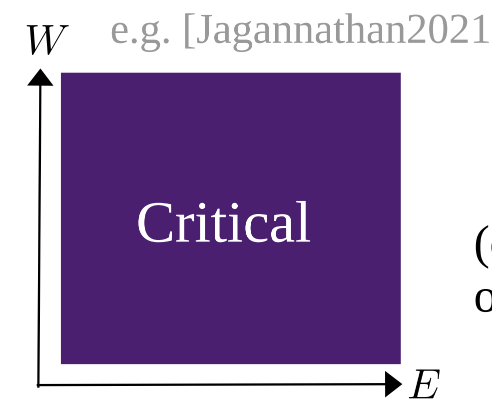

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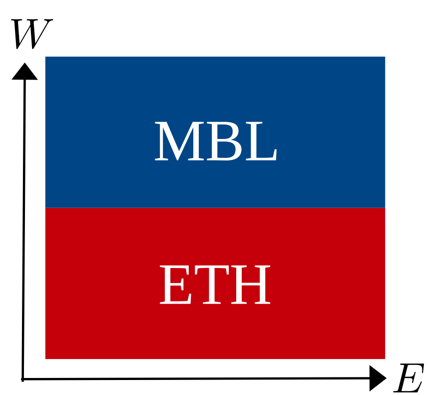

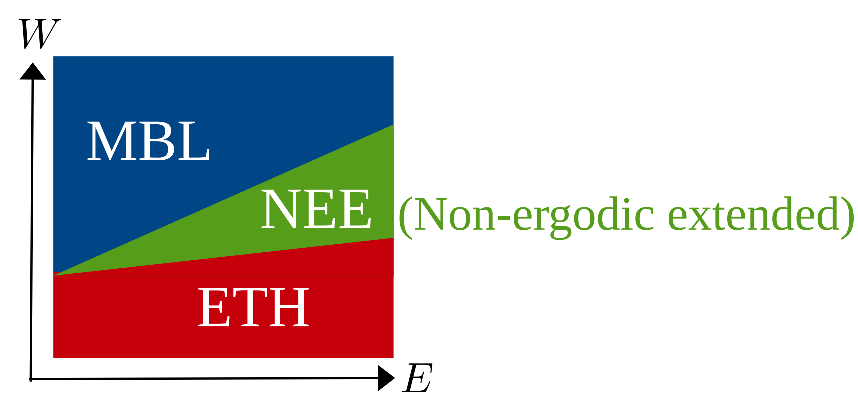

[Li2015, Hsu2018]

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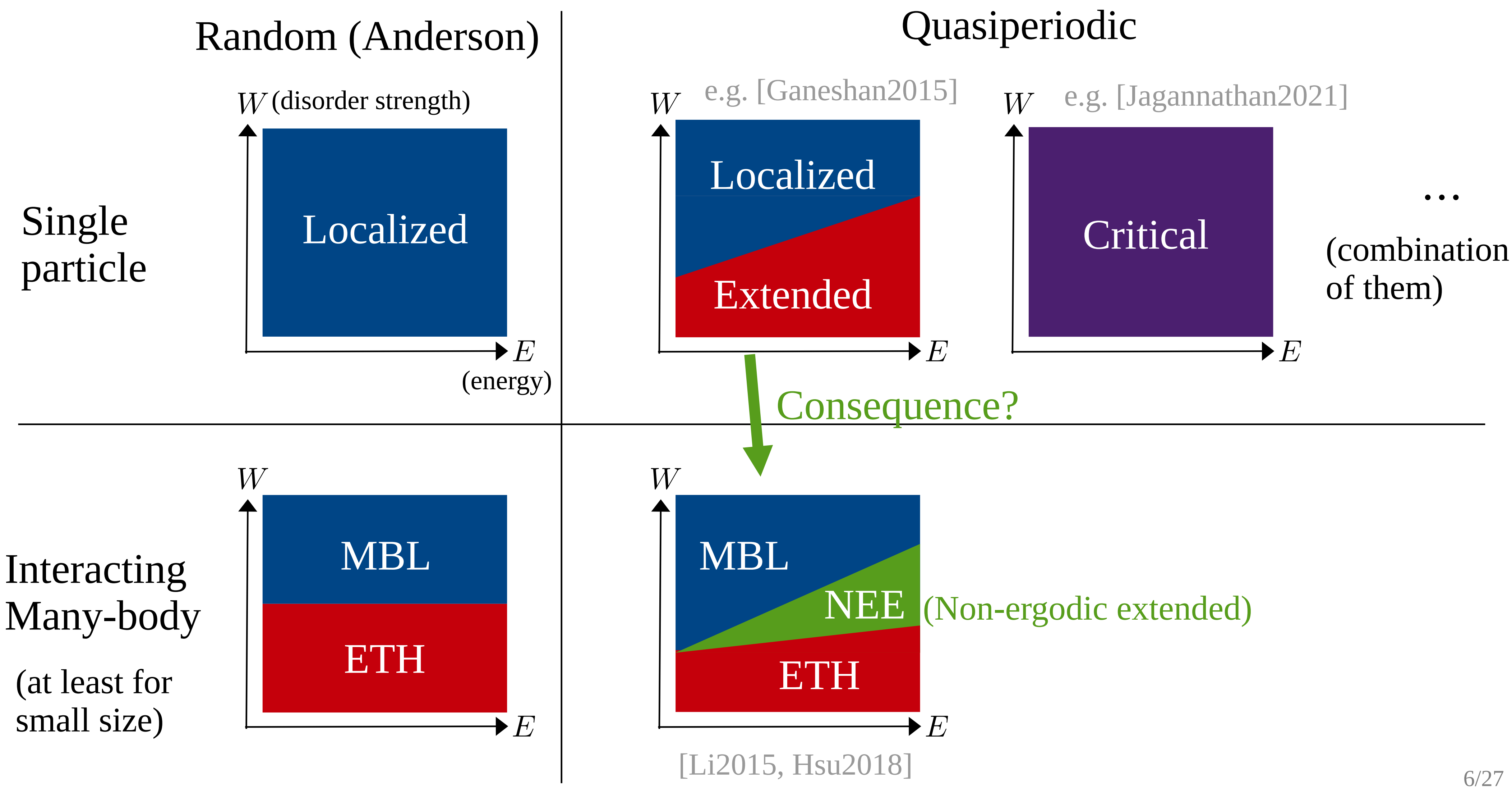

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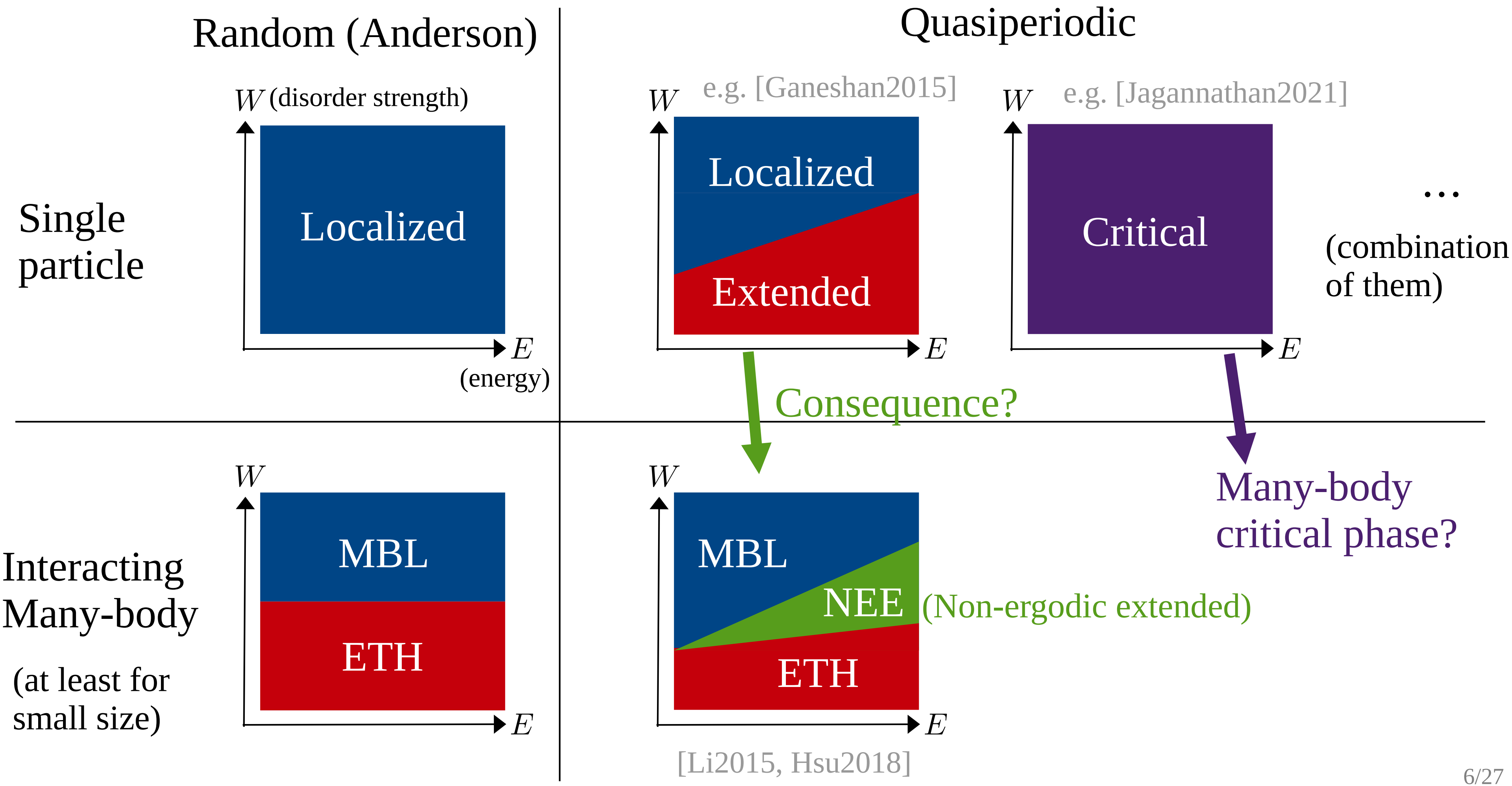
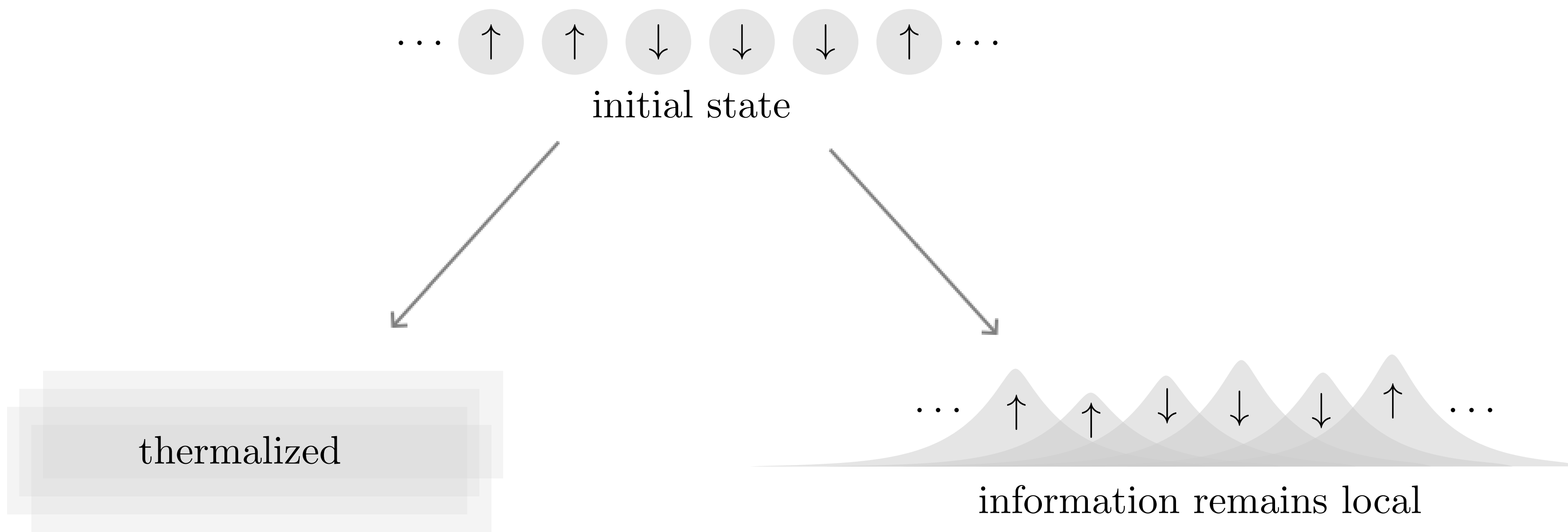
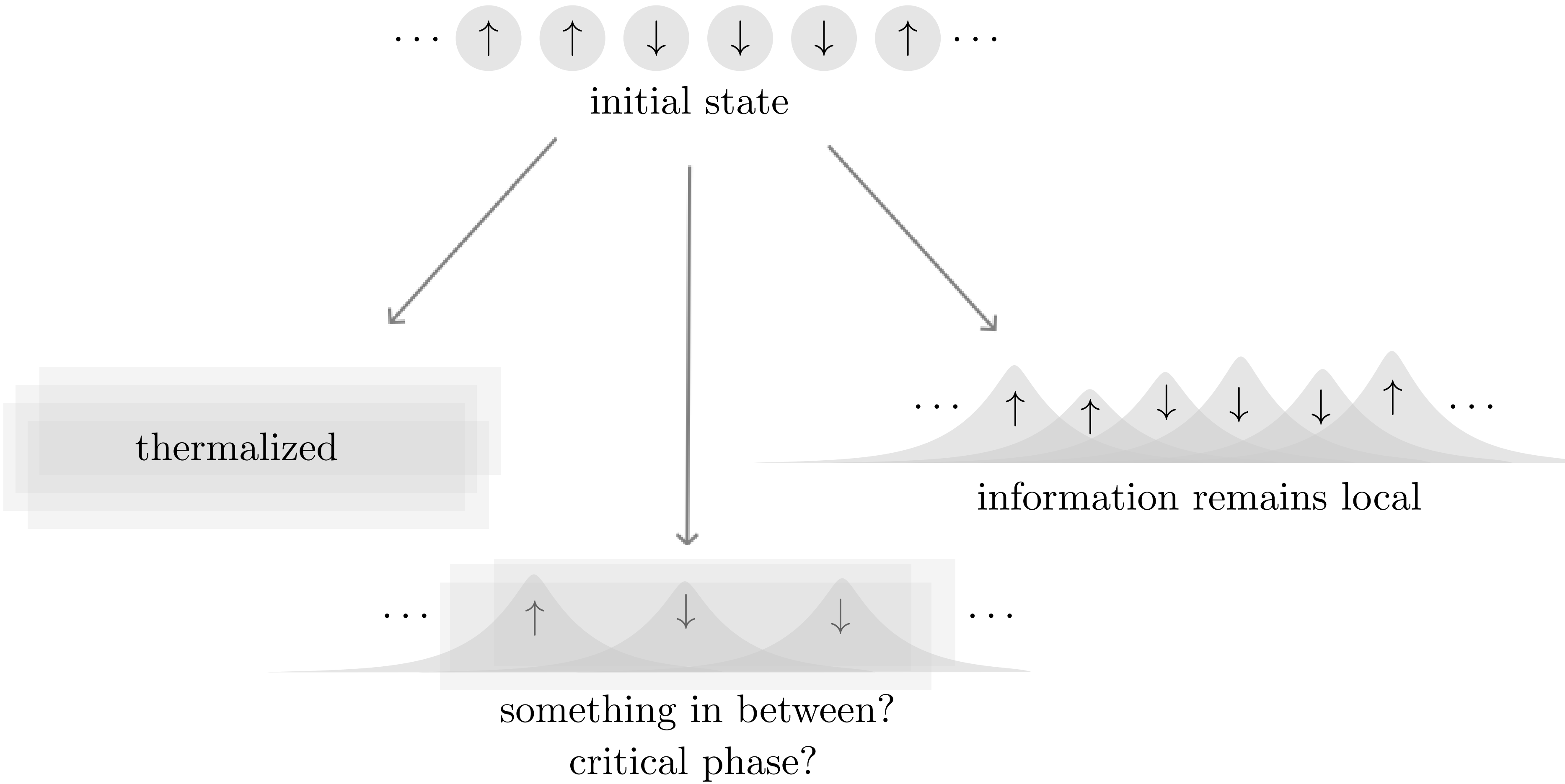




initial state
thermalized
information remains local



initial state
thermalized
information remains local
something in between?
critical phase?

Outline

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  - Many-body localization (MBL)
  - Random v.s. Quasiperiodic systems
- Non-ergodic extended (NEE) regime
  - Numerical results
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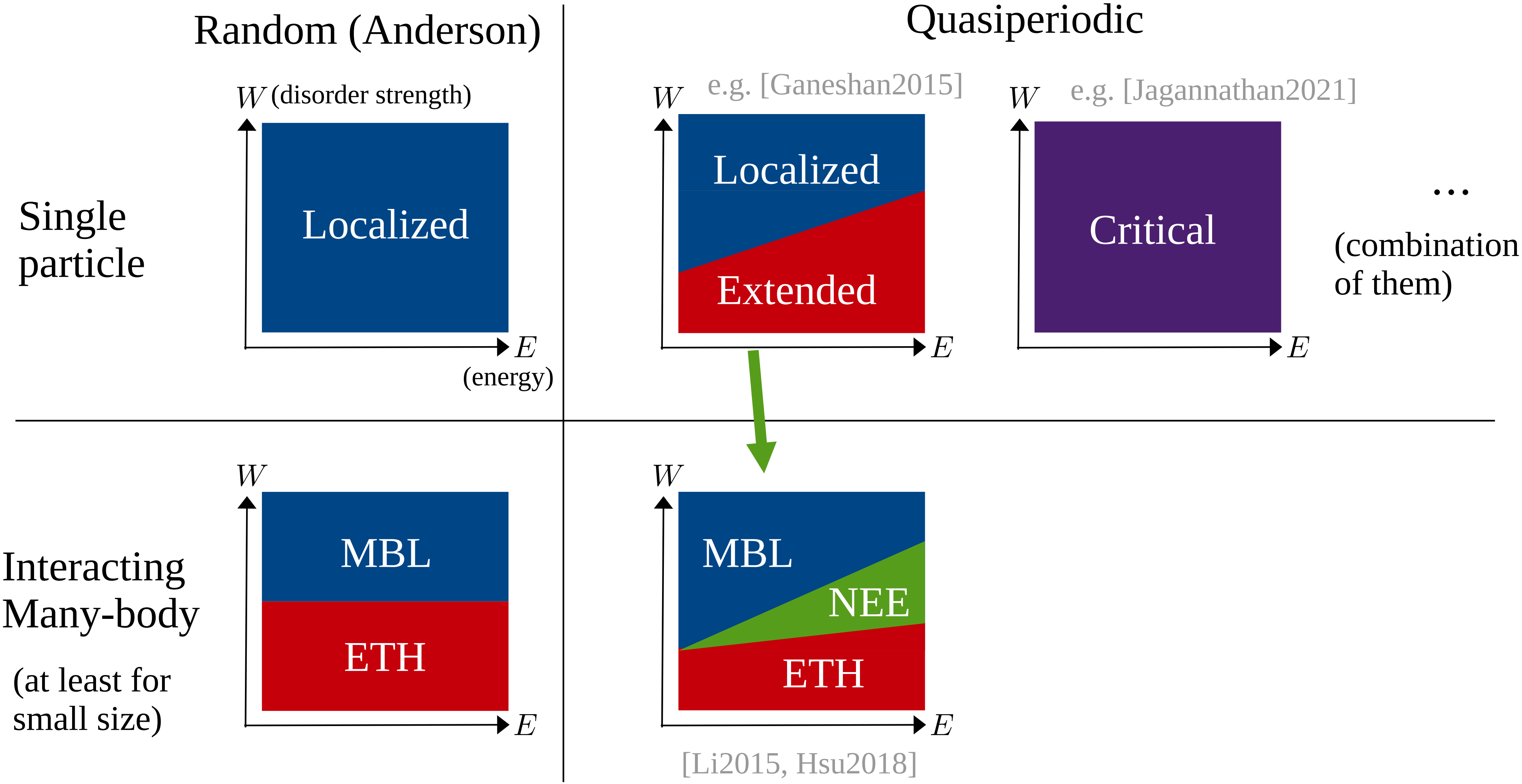

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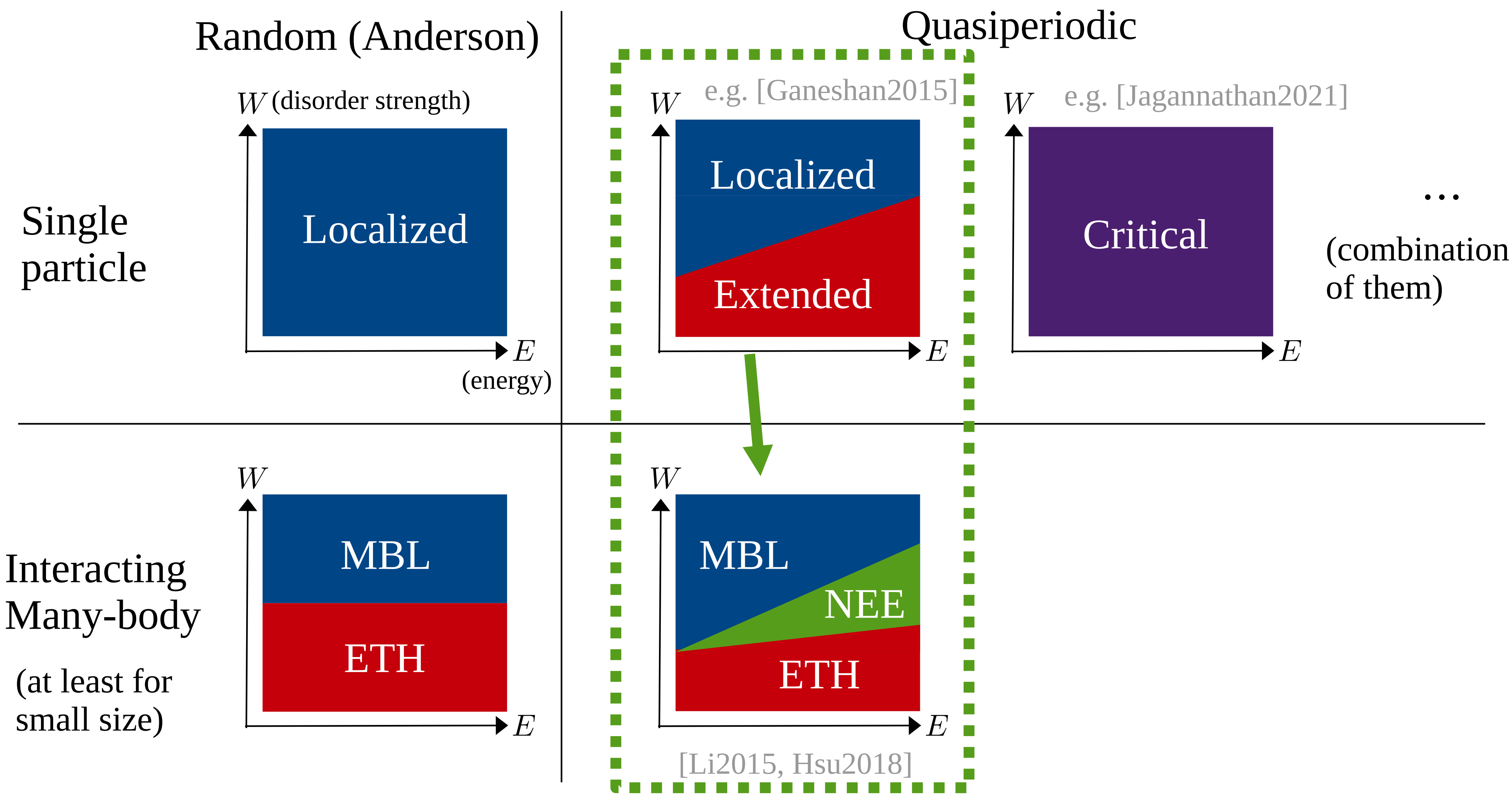

Ganeshan-Pixley-Das Sarma (GPD) model

Ganeshan, Pixley, Das Sarma, PRL (2015)

$$H = \sum_{j=1}^{L-1} \left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + U S_j^z S_{j+1}^z \right) + W \sum_{j=1}^{L} h_j S_j^z, \quad h_j = \frac{\cos(2\pi\varphi j + \phi)}{1 - \alpha \cos(2\pi\varphi j + \phi)}$$

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Spin-1/2

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Interaction strength U=1

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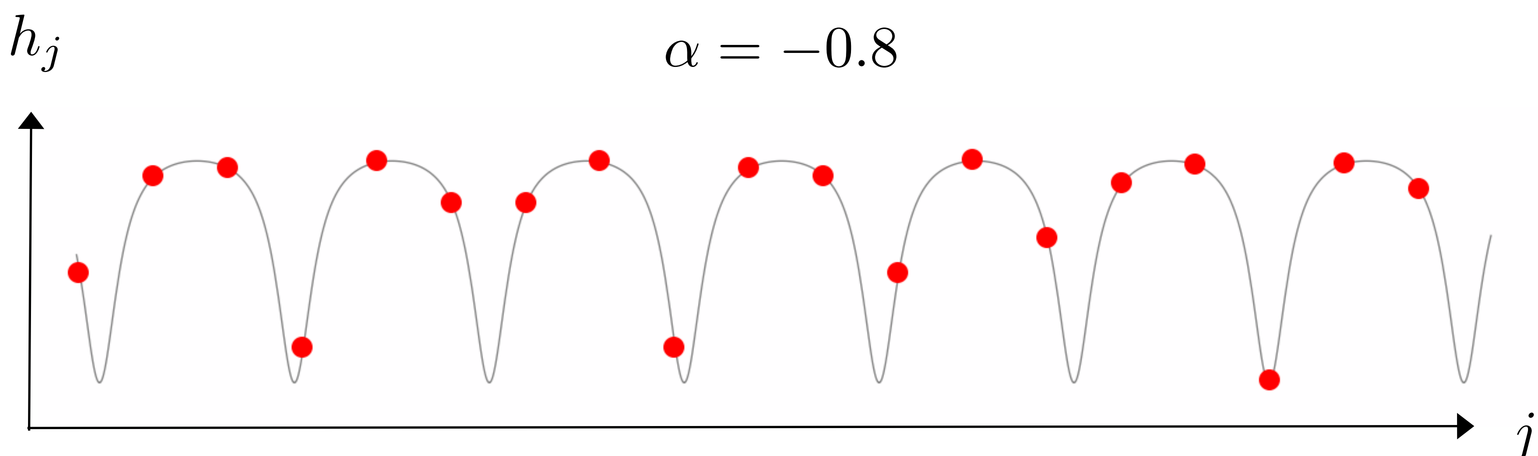

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Non-ergodic extended (NEE) behavior

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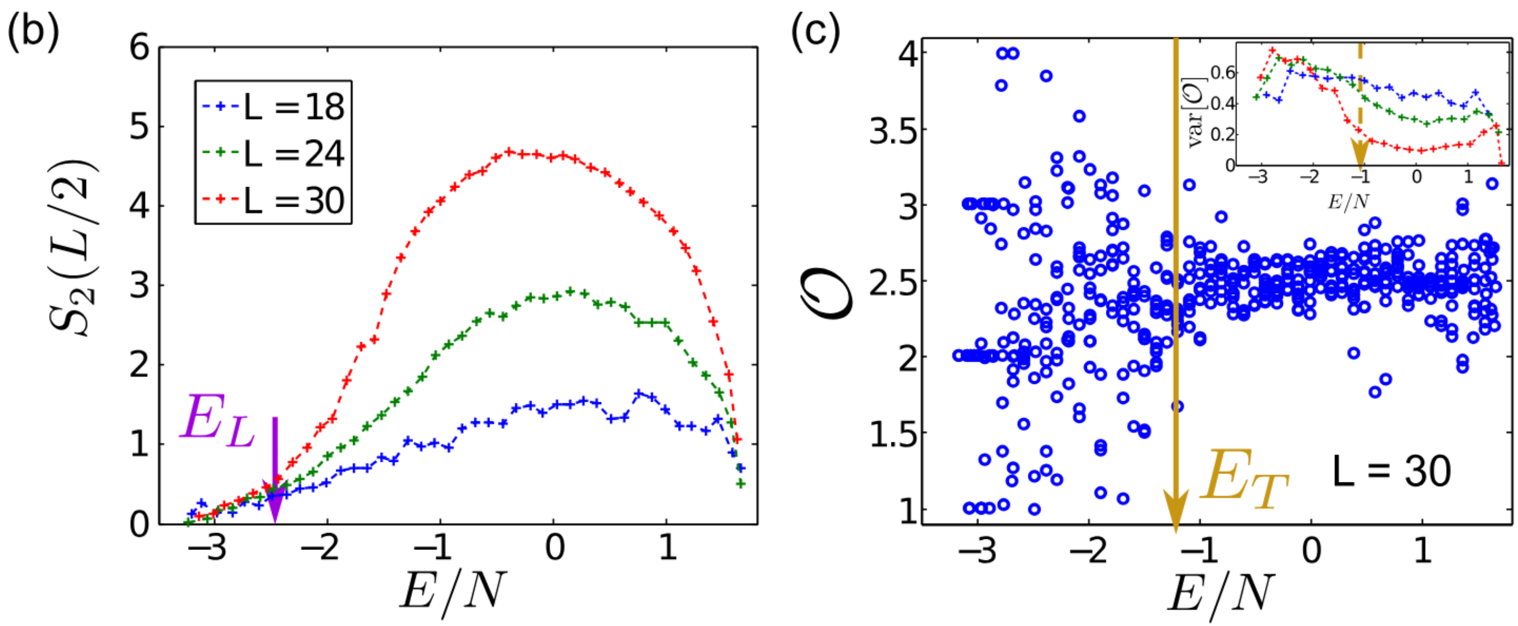

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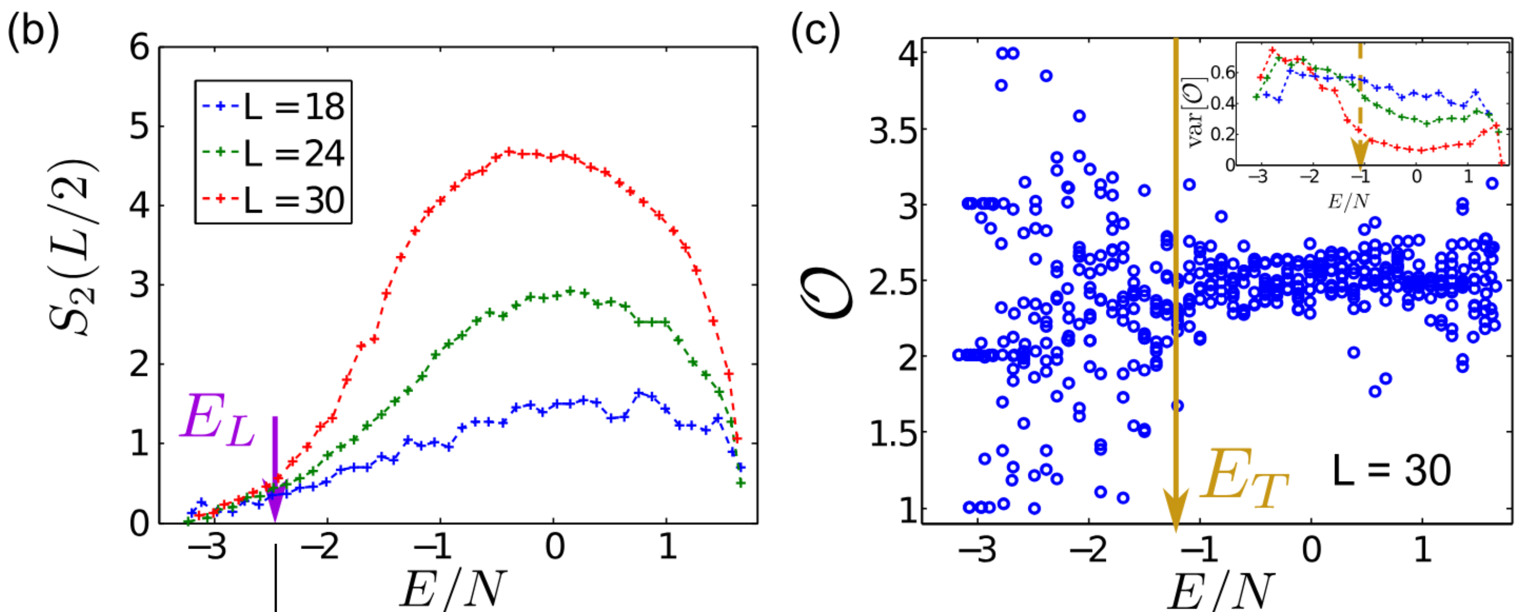

Area-law
(complete LIOMs)

Volume-law entanglement
("extended")

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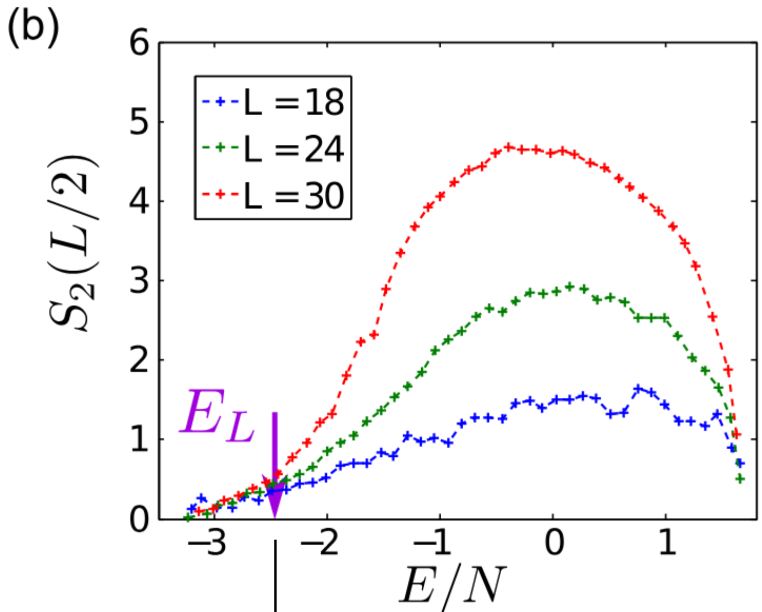

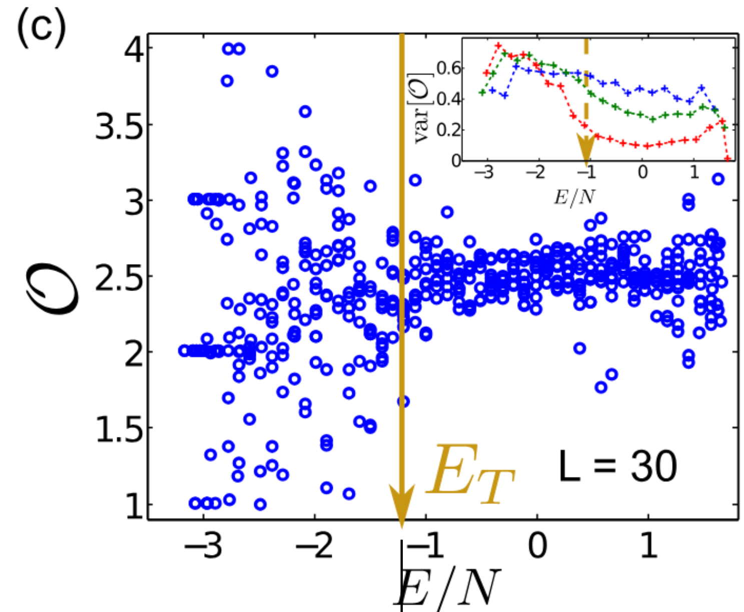

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Volume-law entanglement ("extended")

Violates ETH ("non-ergodic")

Satisfies ETH

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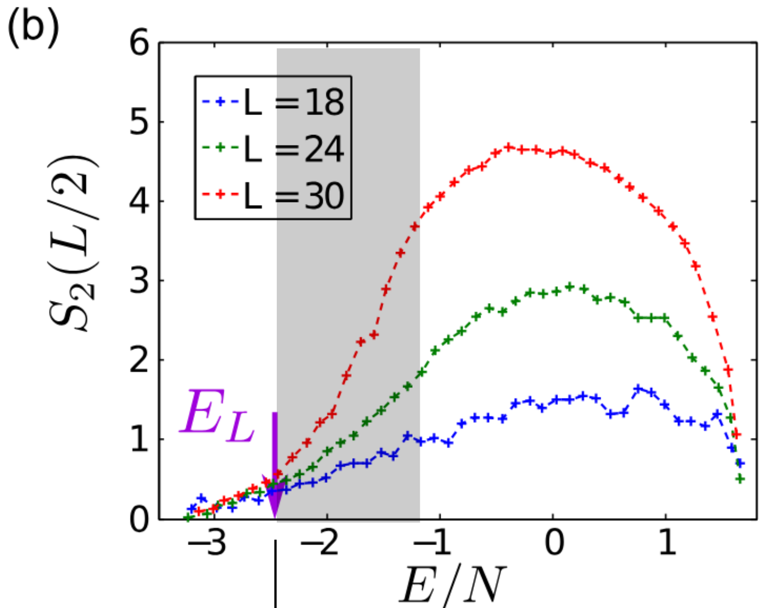

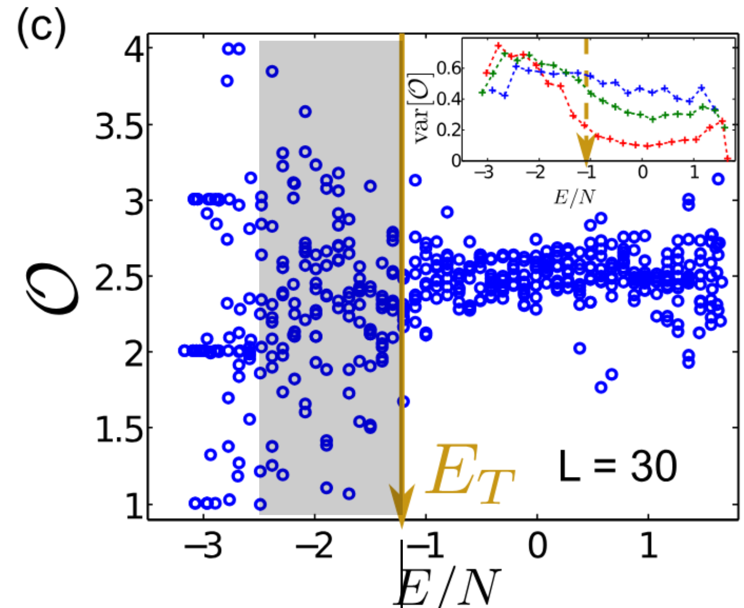

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Spin-spin correlator in the prethermal regime

$$C(E,t) = \left[\langle \psi_E | S^z_{L/2}(t) S^z_{L/2}(0) | \psi_E \rangle\right] - \left[\langle \psi_E | S^z_{L/2}(0) | \psi_E \rangle\right]^2$$

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Haar-random superposition
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Long et al. PRL '23; Long et al. SciPost '23

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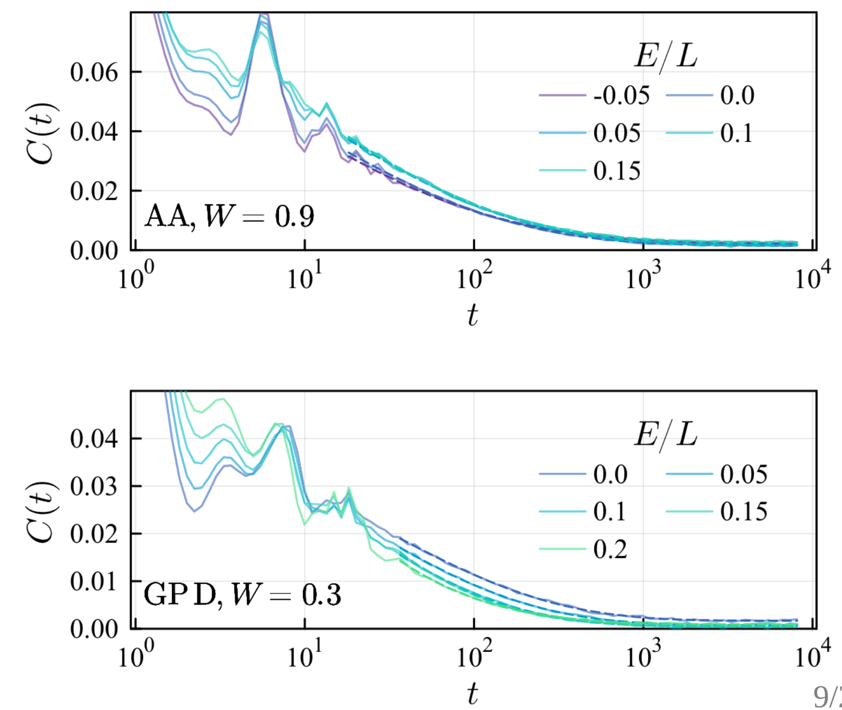
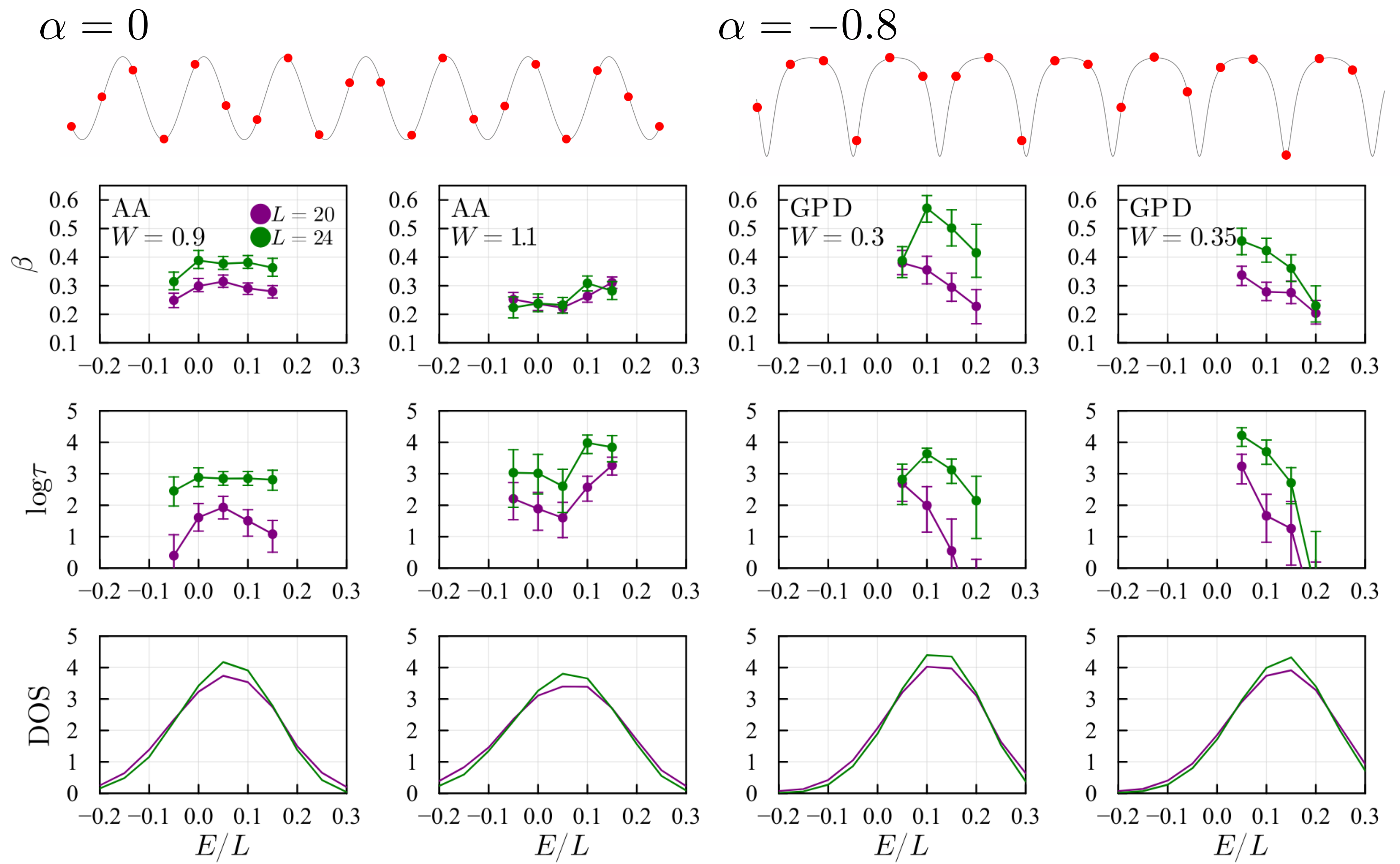


Stretched exponential fit

YTT, Long, Das Sarma PRB (2024)



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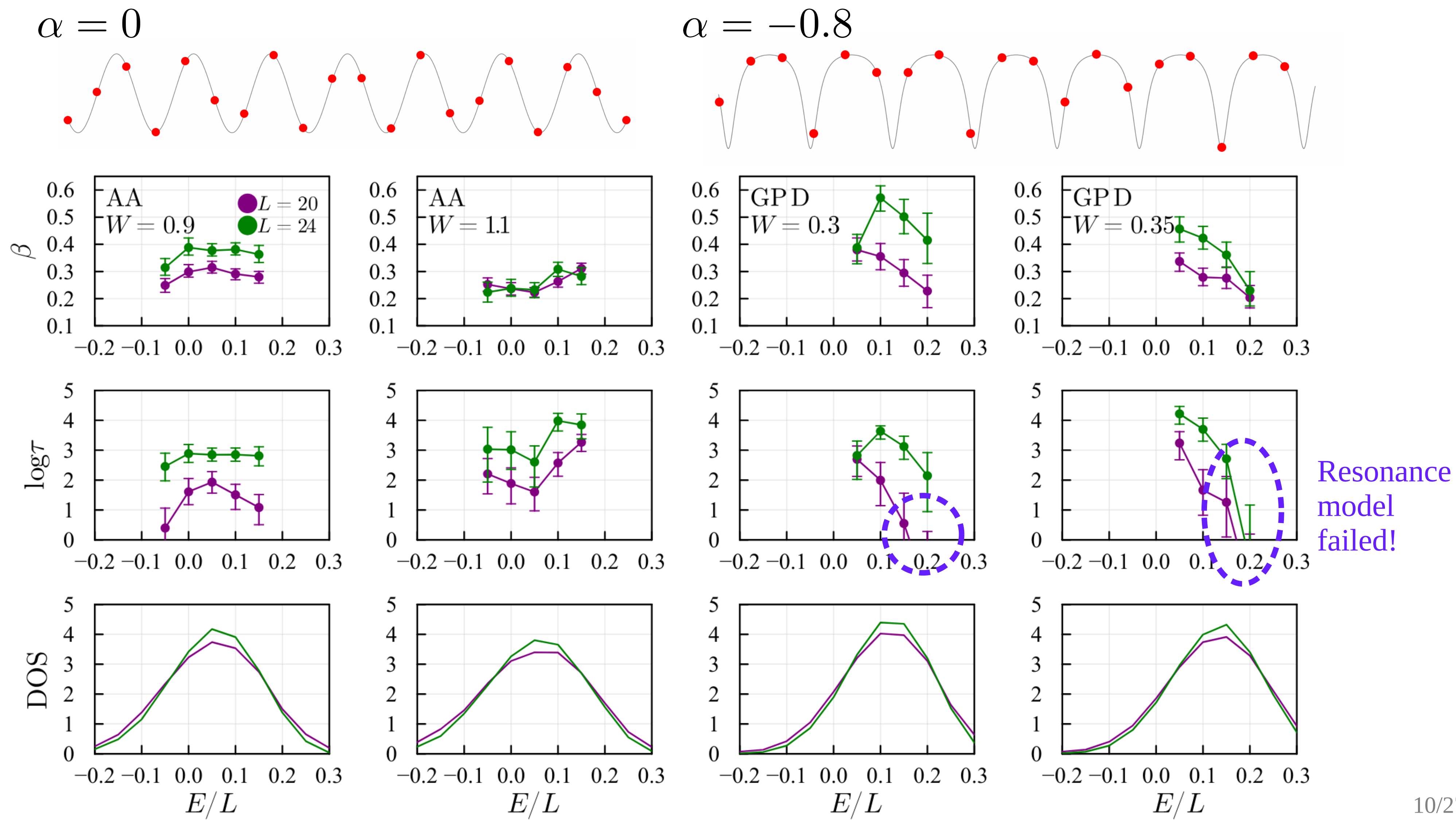

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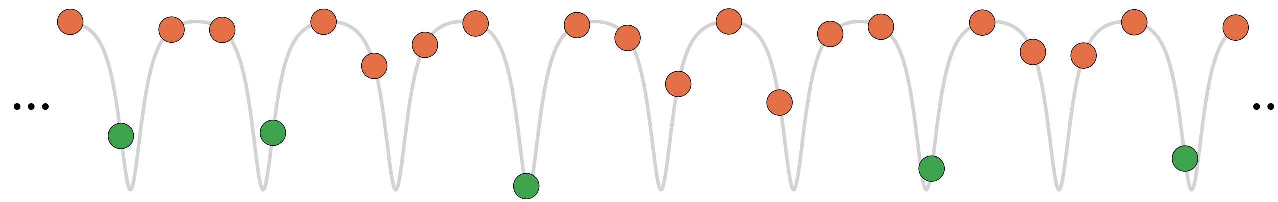
Effects of regularly spaced deep wells

YTT, Long, Das Sarma PRB (2024)

GPD potential:



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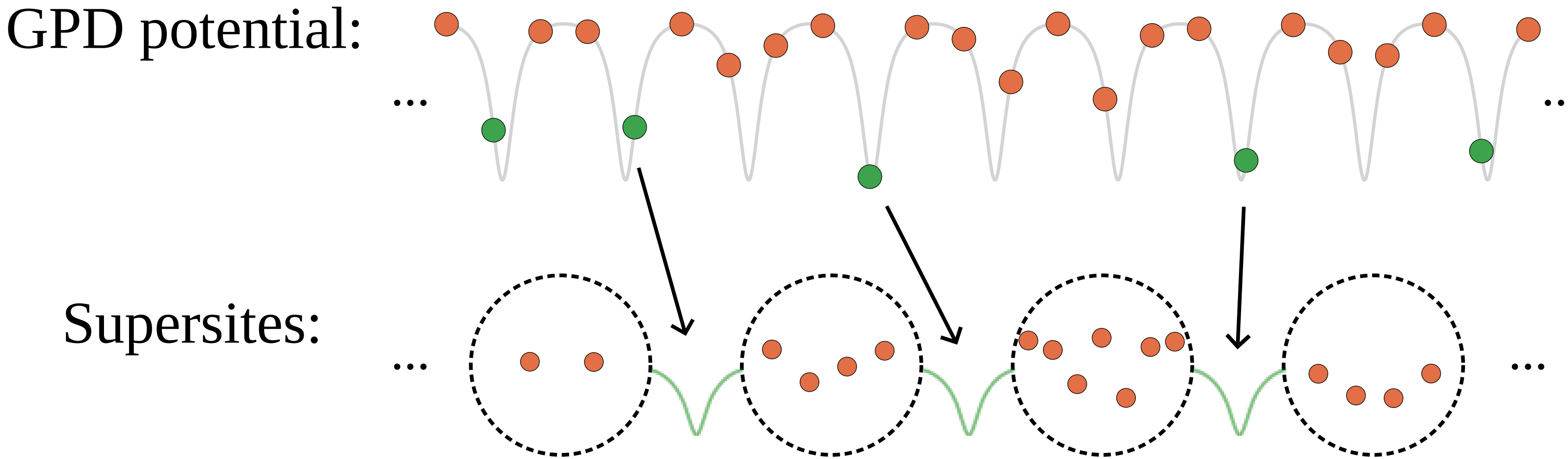

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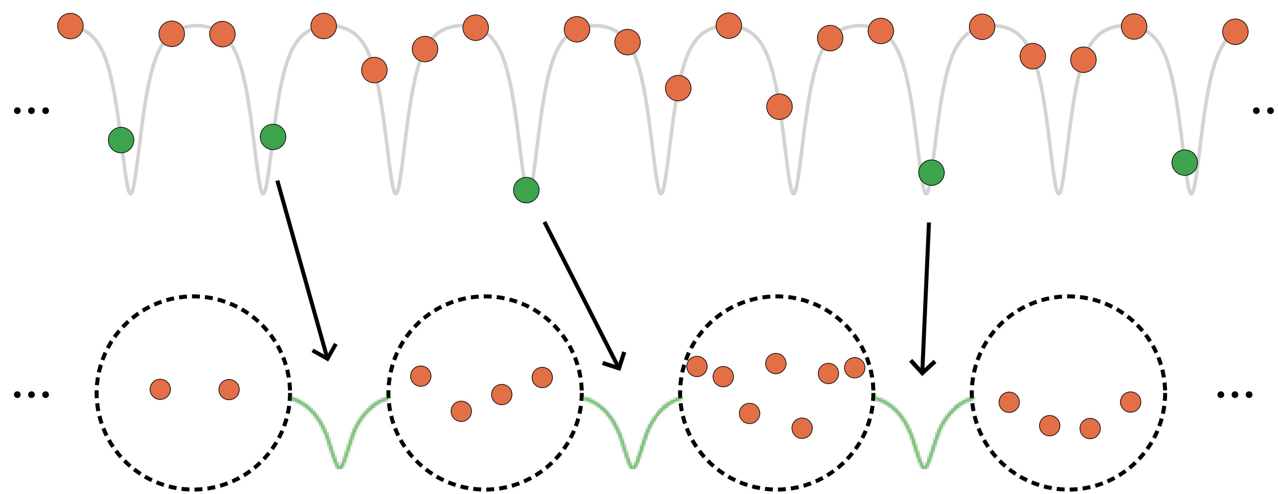
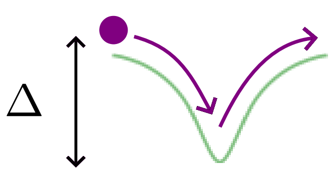


Inter-supersite hopping:



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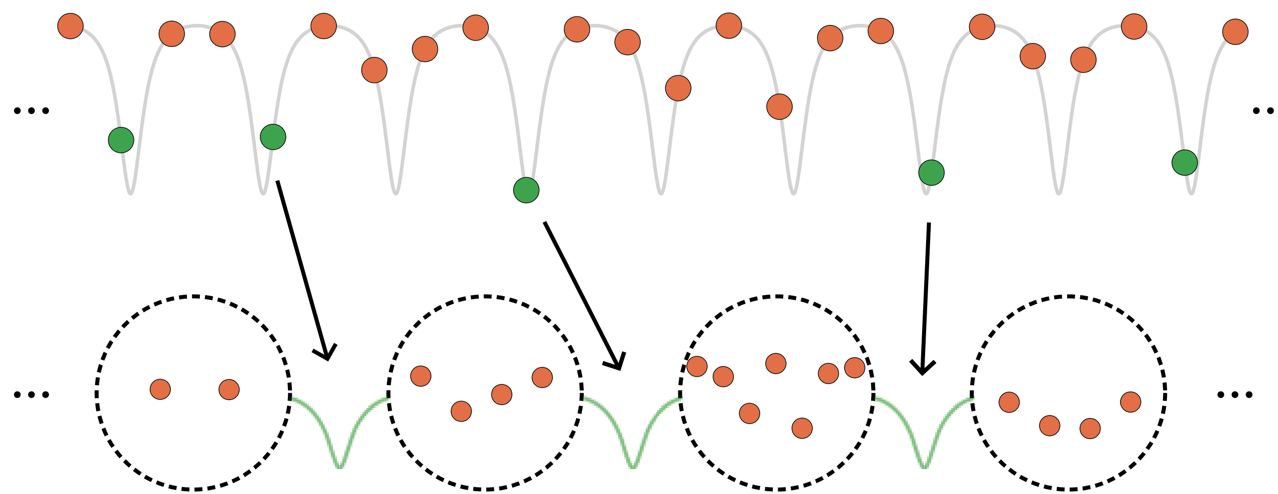

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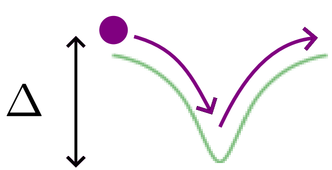

$$\sim \frac{J^2}{\Delta}$$

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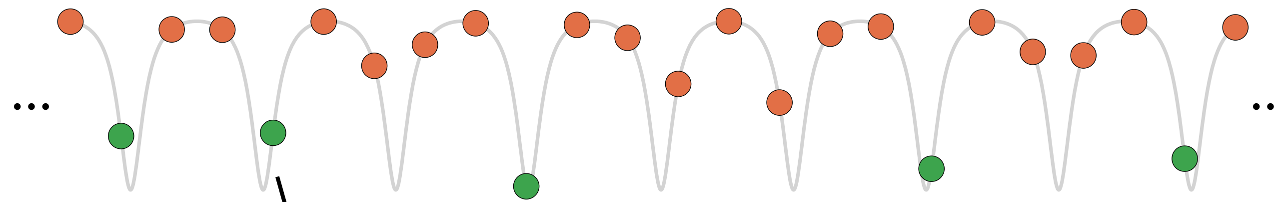

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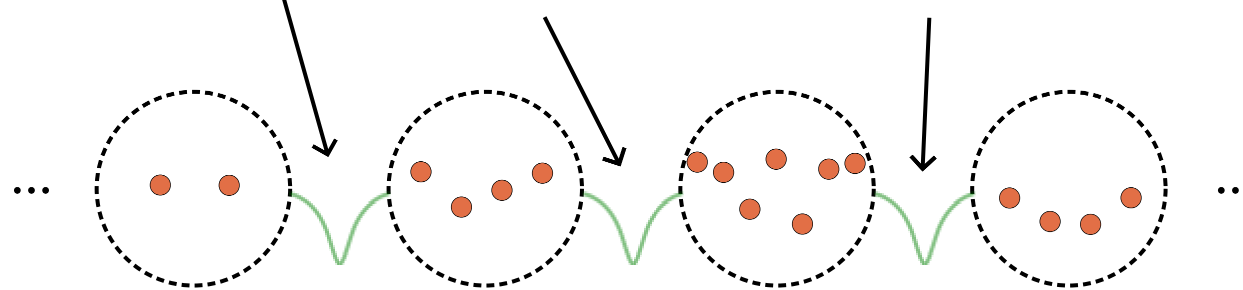

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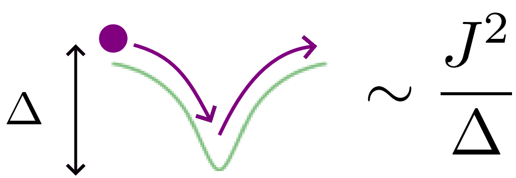

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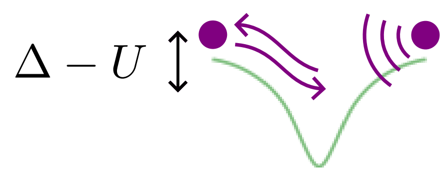

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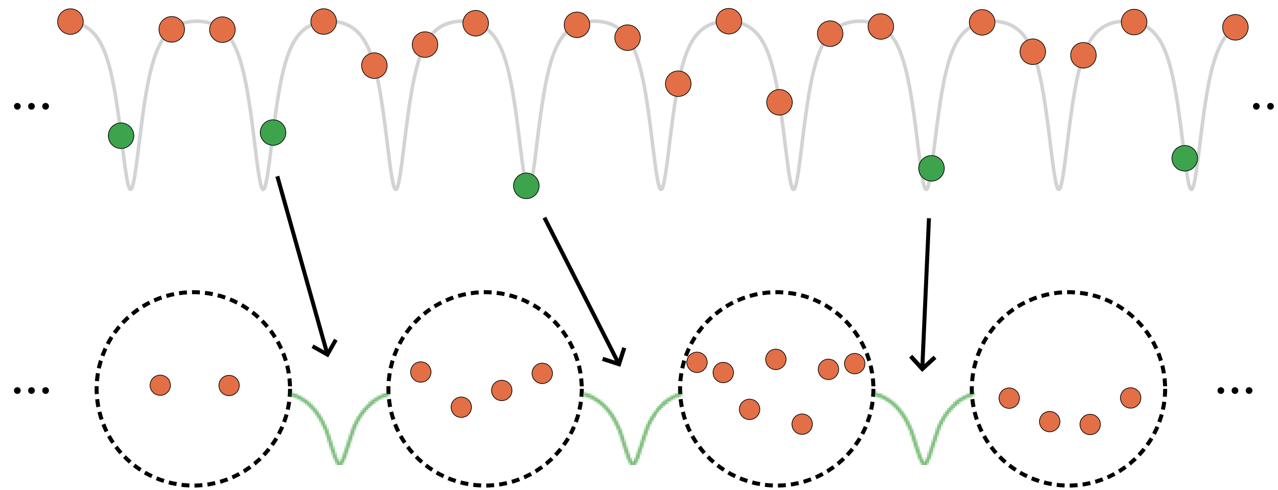

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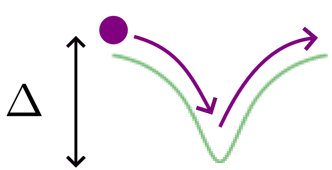

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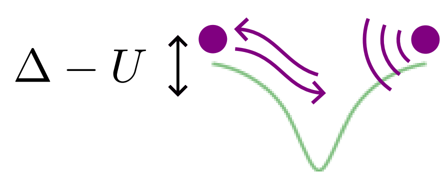

$$\sim \frac{J^2}{\Delta - U} - \frac{J^2}{\Delta}$$

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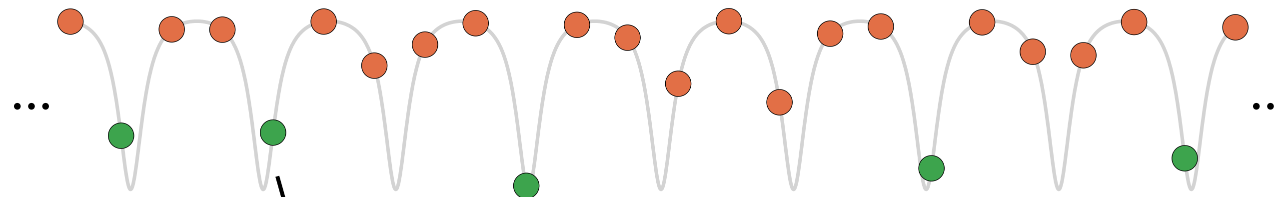

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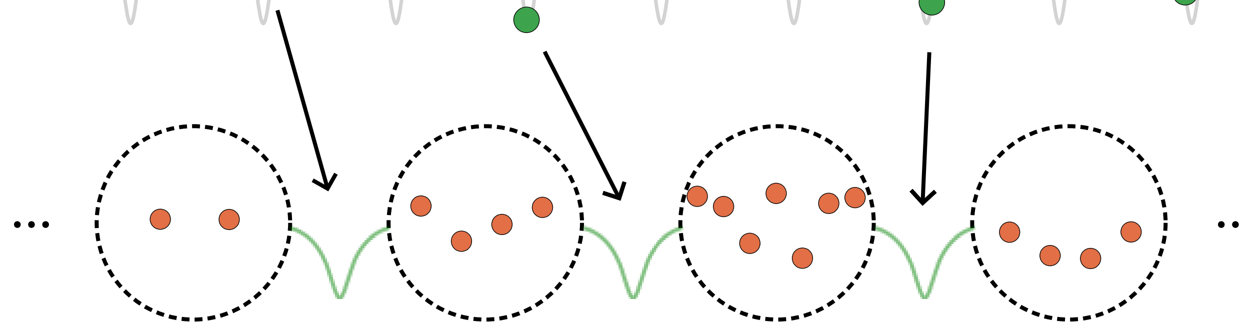

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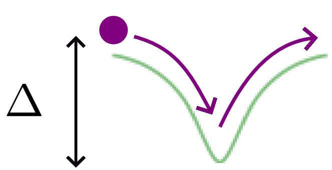

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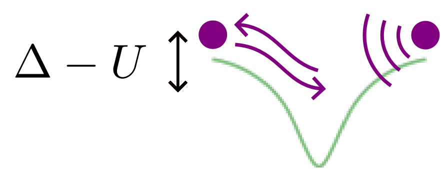


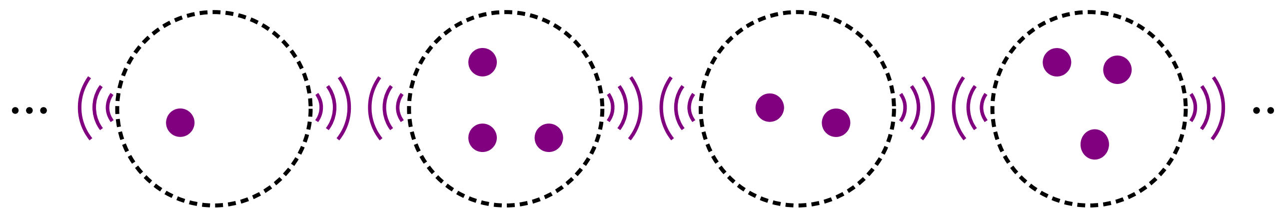
$$\sim \frac{J^2}{\Delta - U} - \frac{J^2}{\Delta}$$

Can be much larger!

Non-ergodic extended (NEE) behavior

YTT, Long, Das Sarma PRB (2024)

Approximate inter-supersite hopping as 0 but interaction being finite.



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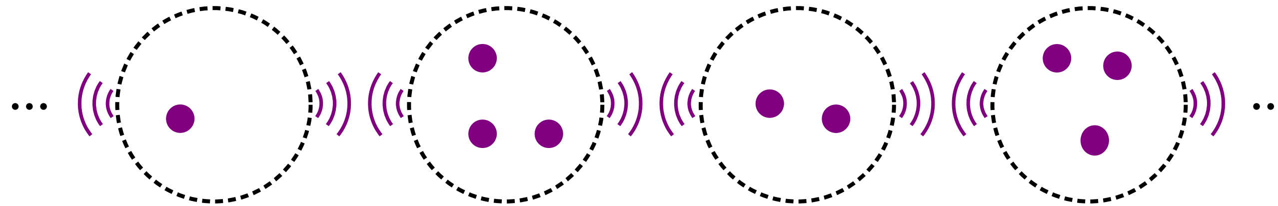

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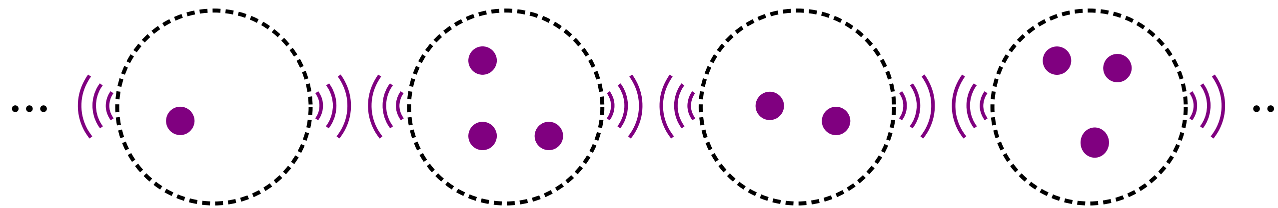

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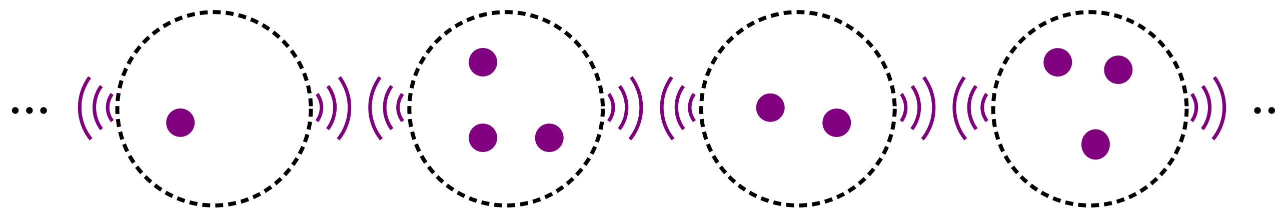

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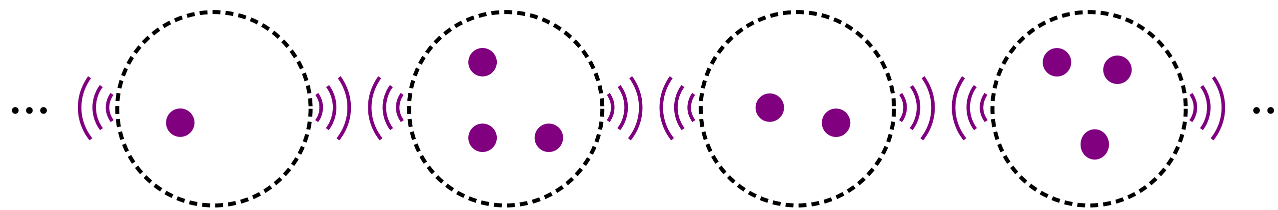

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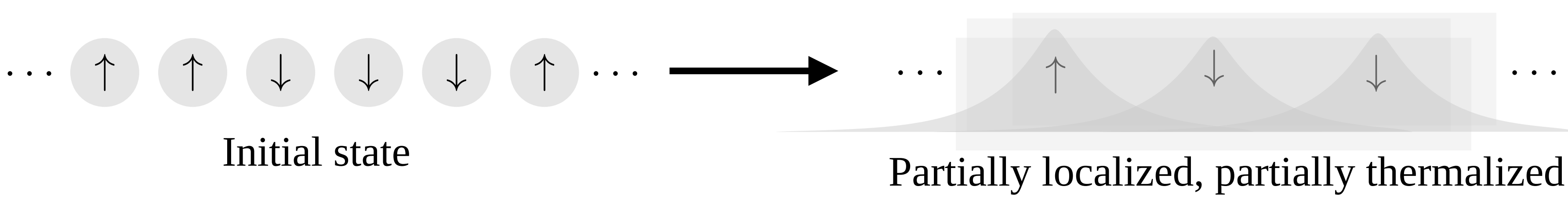

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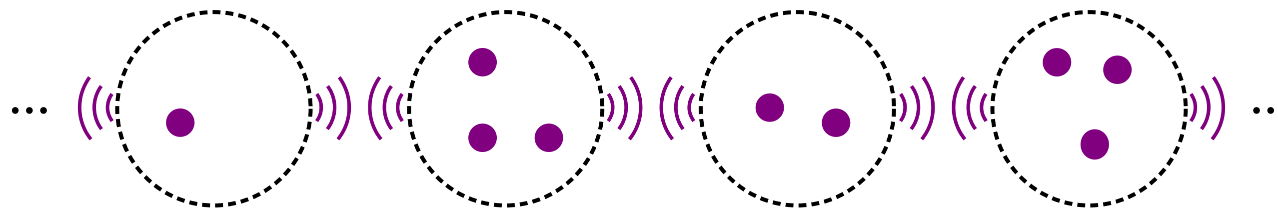

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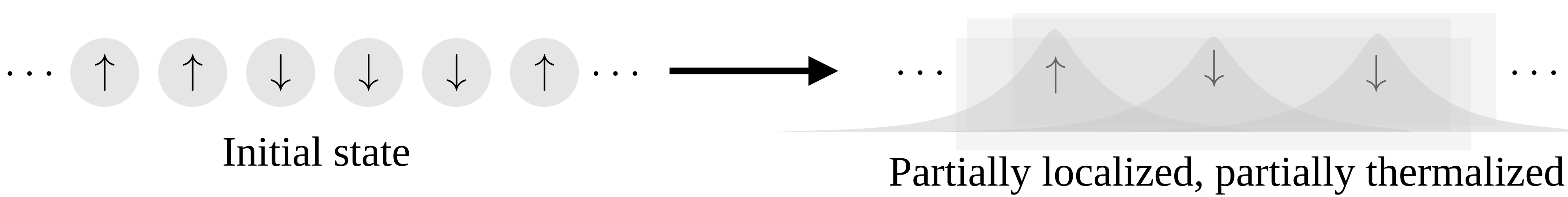
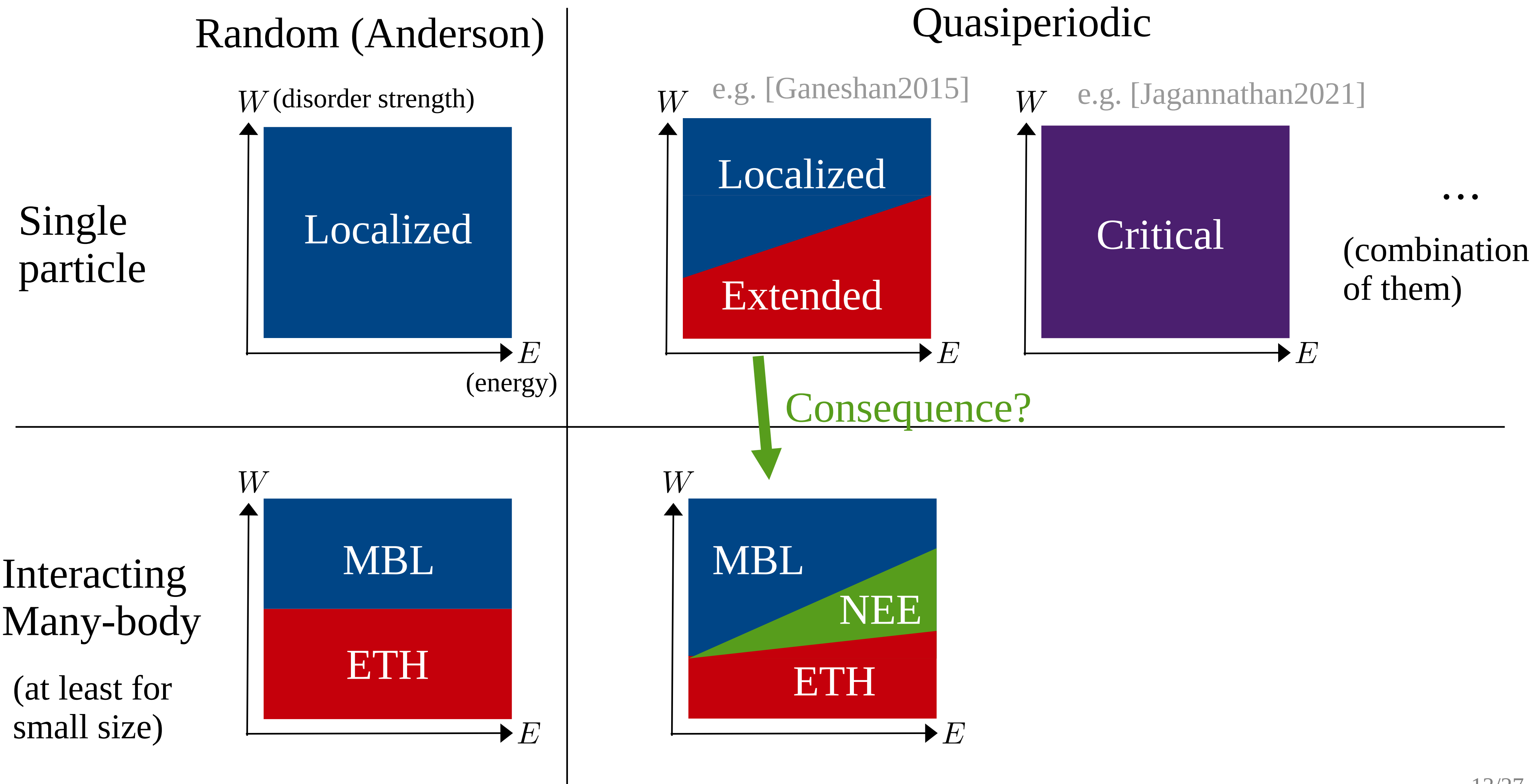
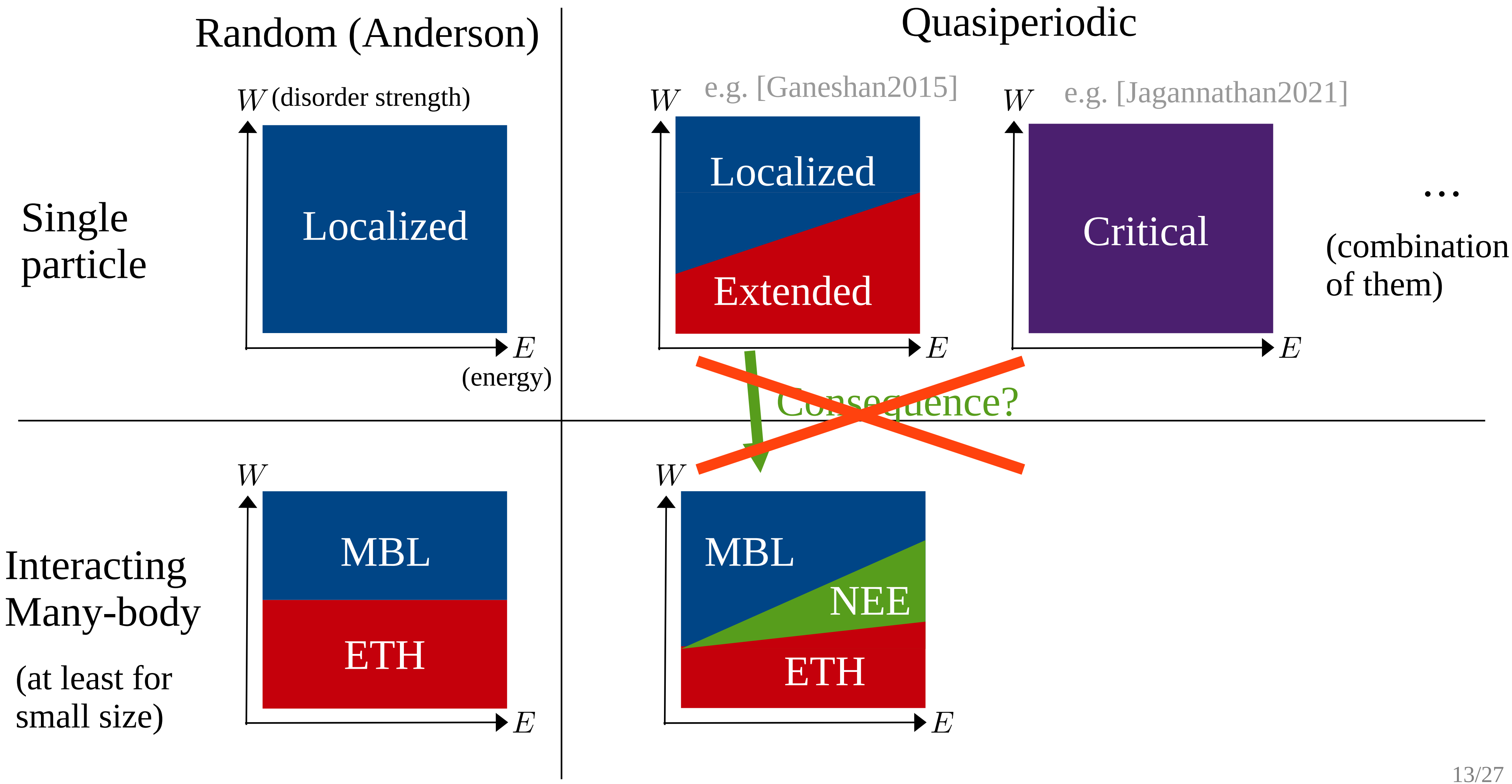


Expected to be a finite-size/time behavior (eventually everything thermalizes)

What’s special about 1D quasiperiodic systems



What’s special about 1D quasiperiodic systems



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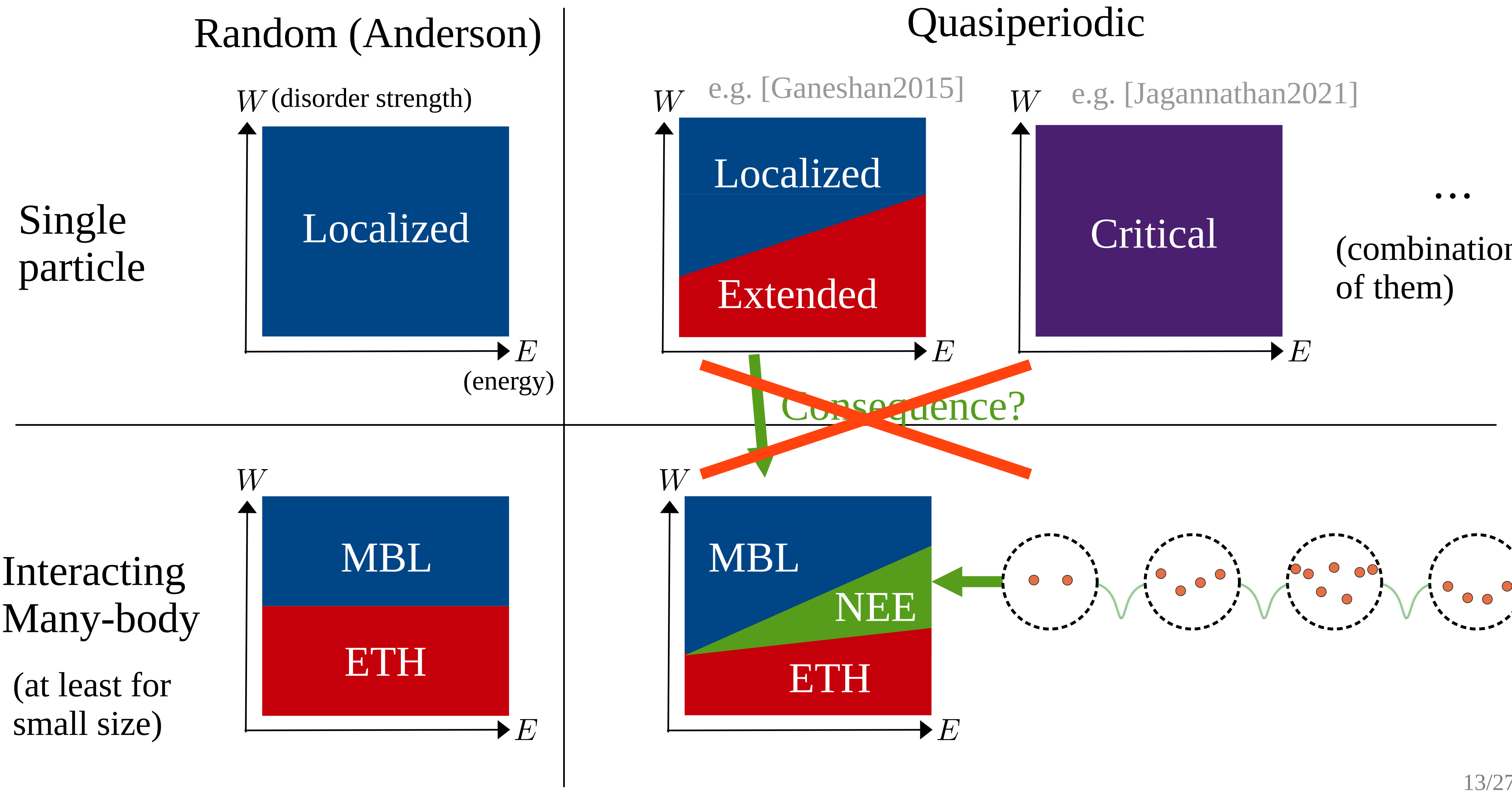
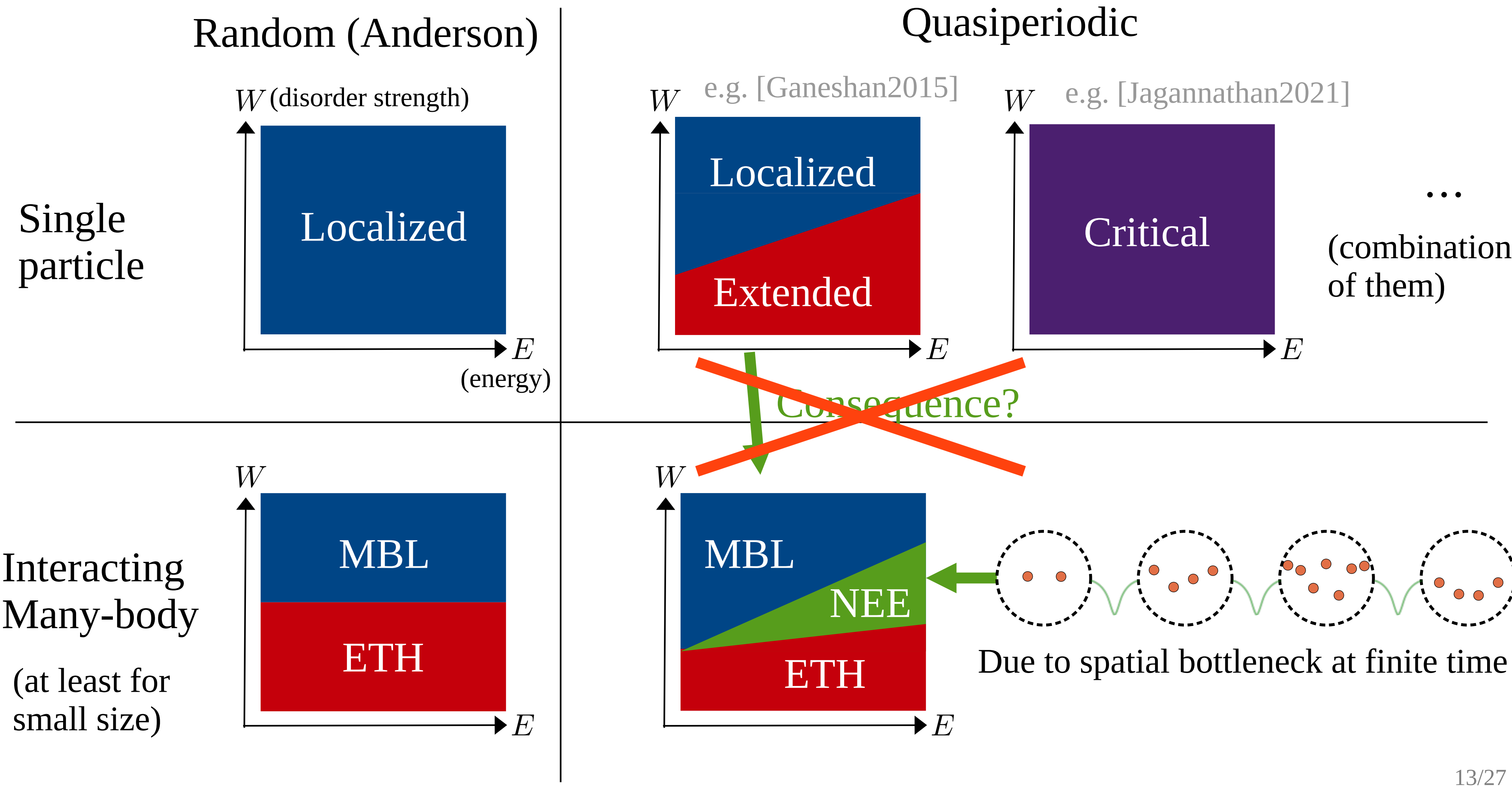


What's special about 1D quasiperiodic systems



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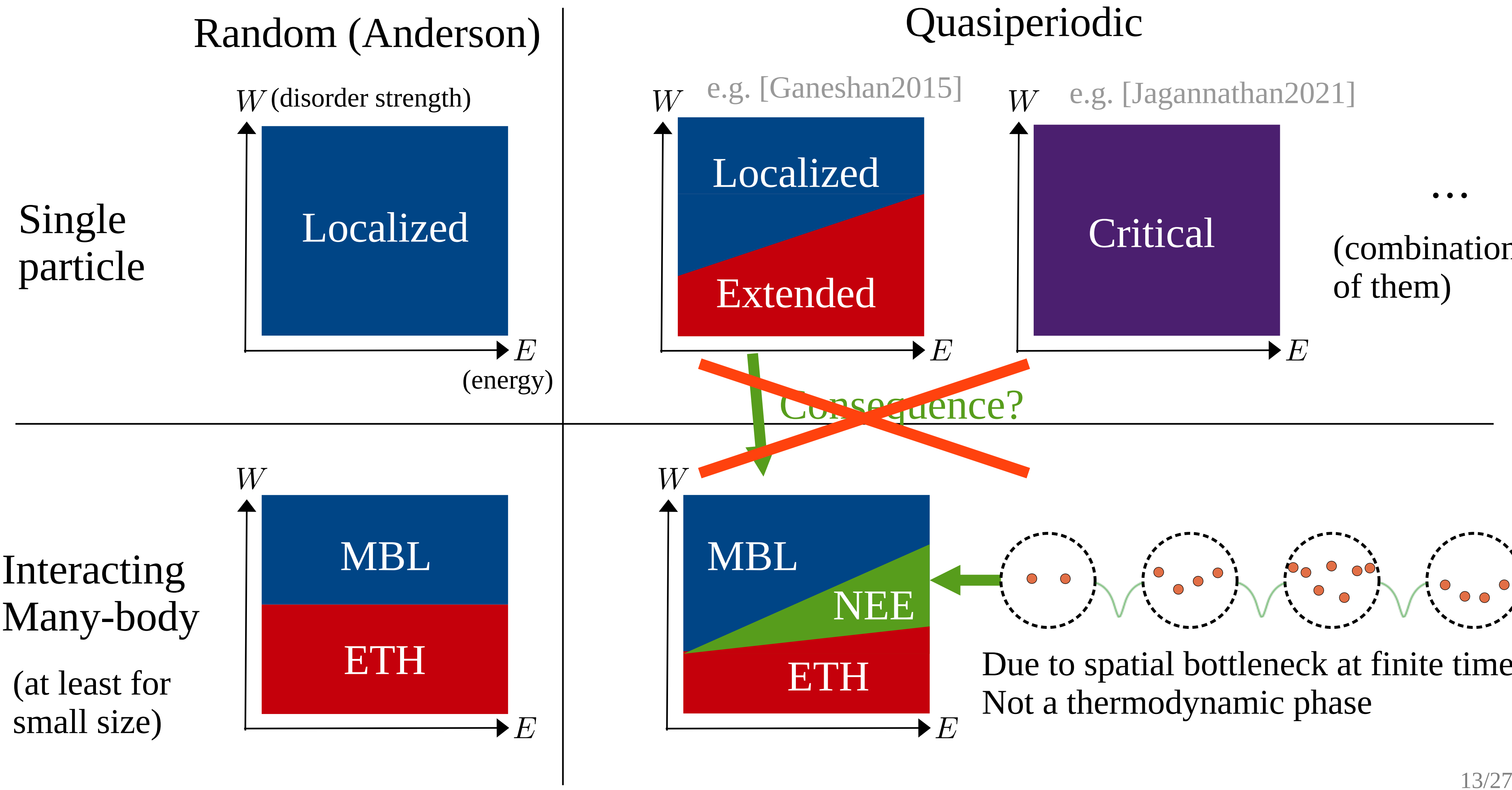

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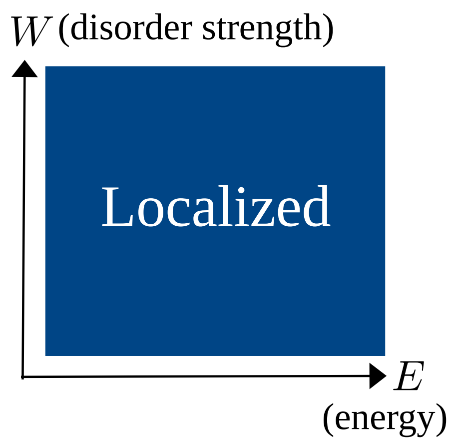
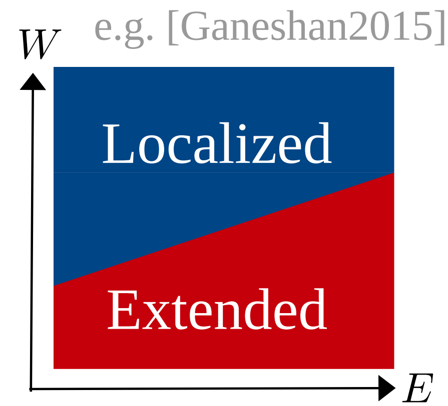
What's special about 1D quasiperiodic systems

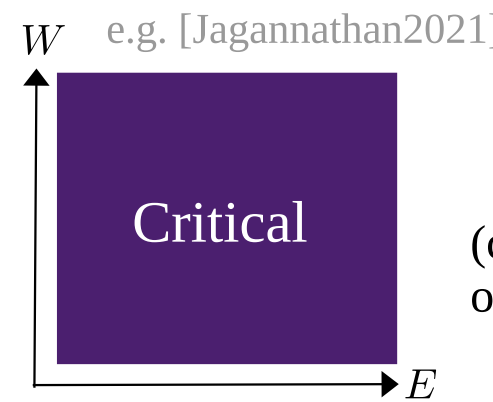
Random (Anderson)

Quasiperiodic

Single particle





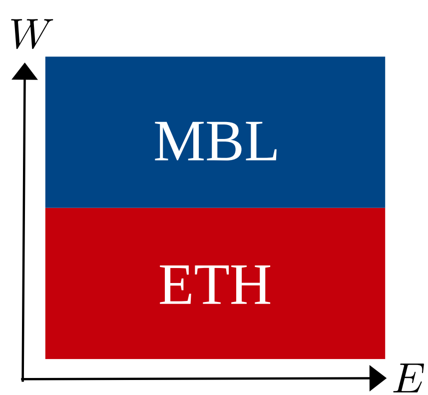


...

(combination of them)

Interacting Many-body

(at least for small size)

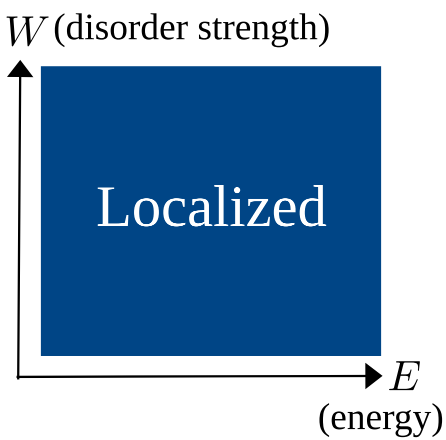


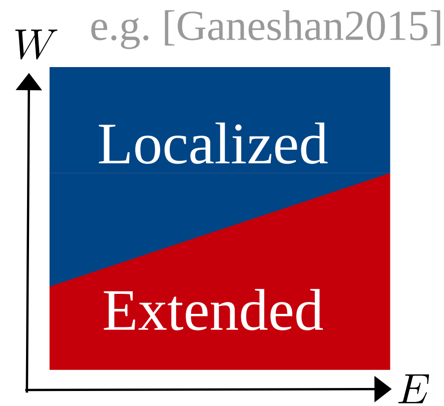
What's special about 1D quasiperiodic systems

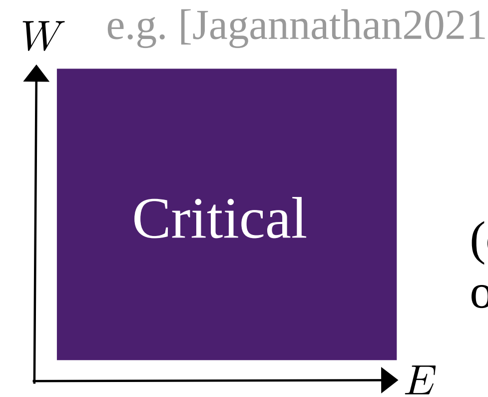
Random (Anderson)

Quasiperiodic

Single particle







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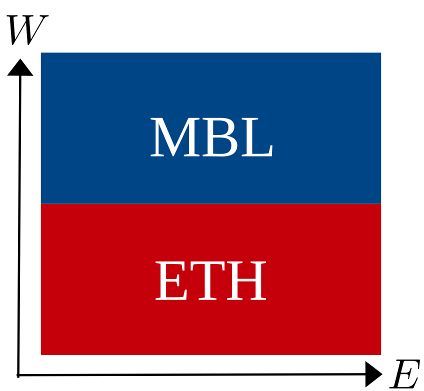

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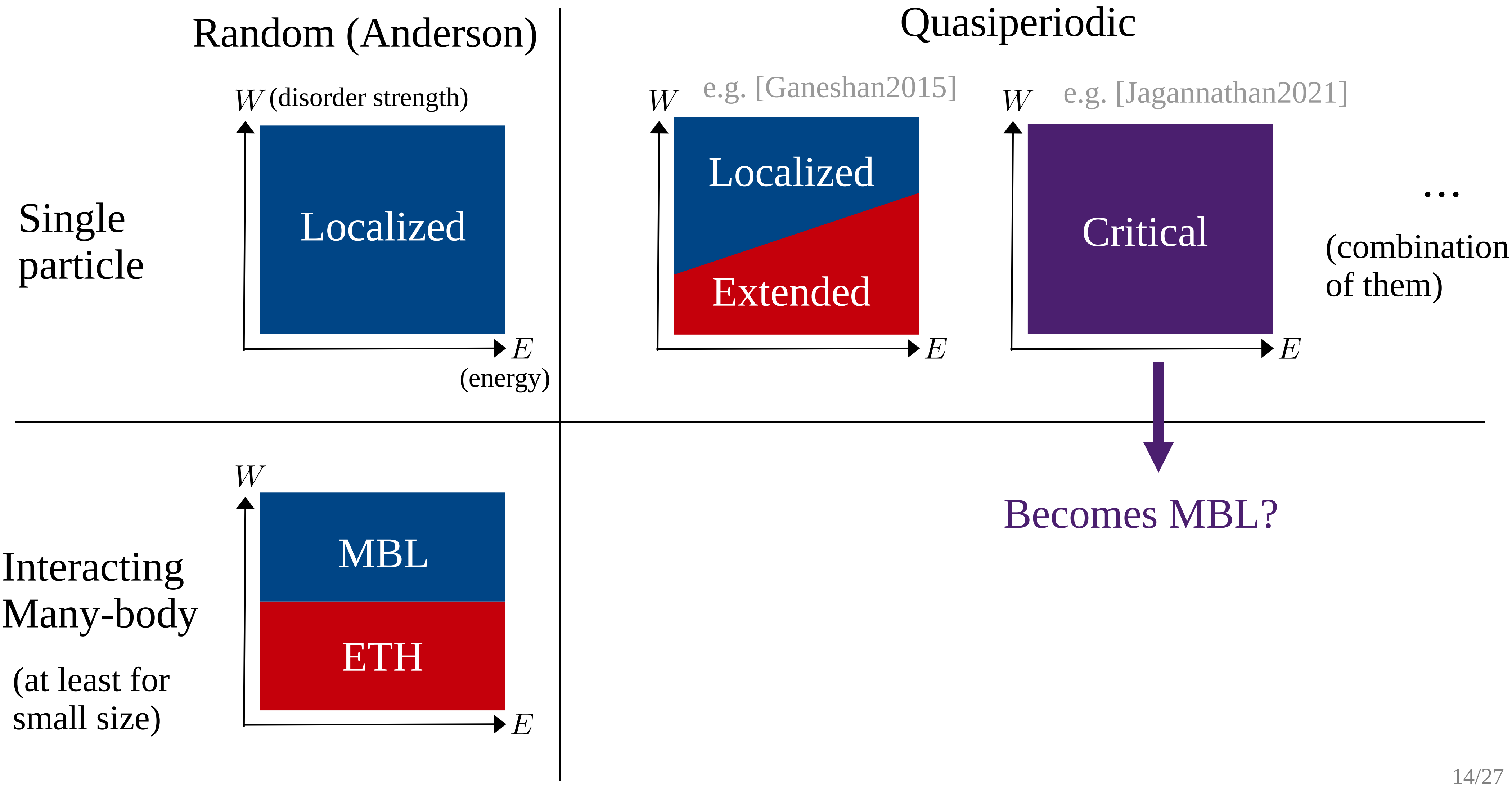

What’s special about 1D quasiperiodic systems

Random (Anderson)

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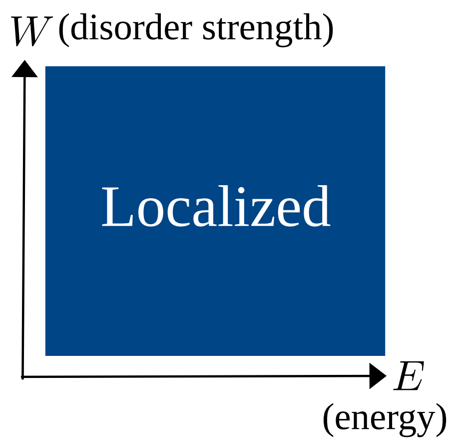
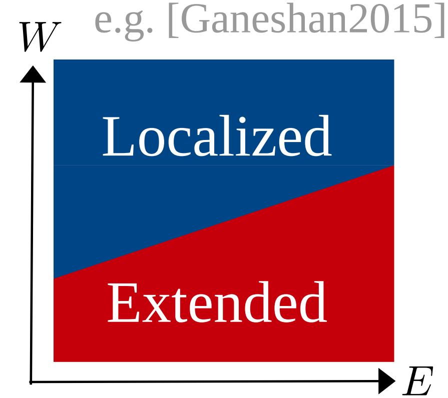
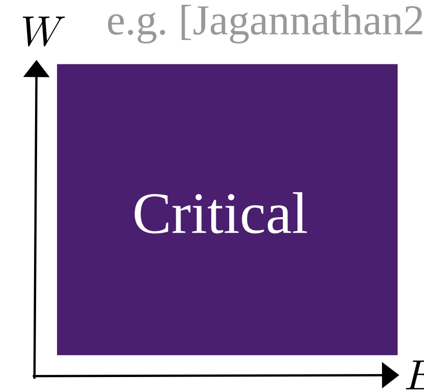


e.g. [Ganeshan2015]



e.g. [Jagannathan2021]

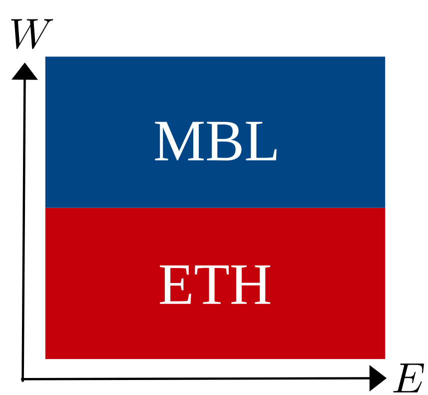


...

(combination of them)

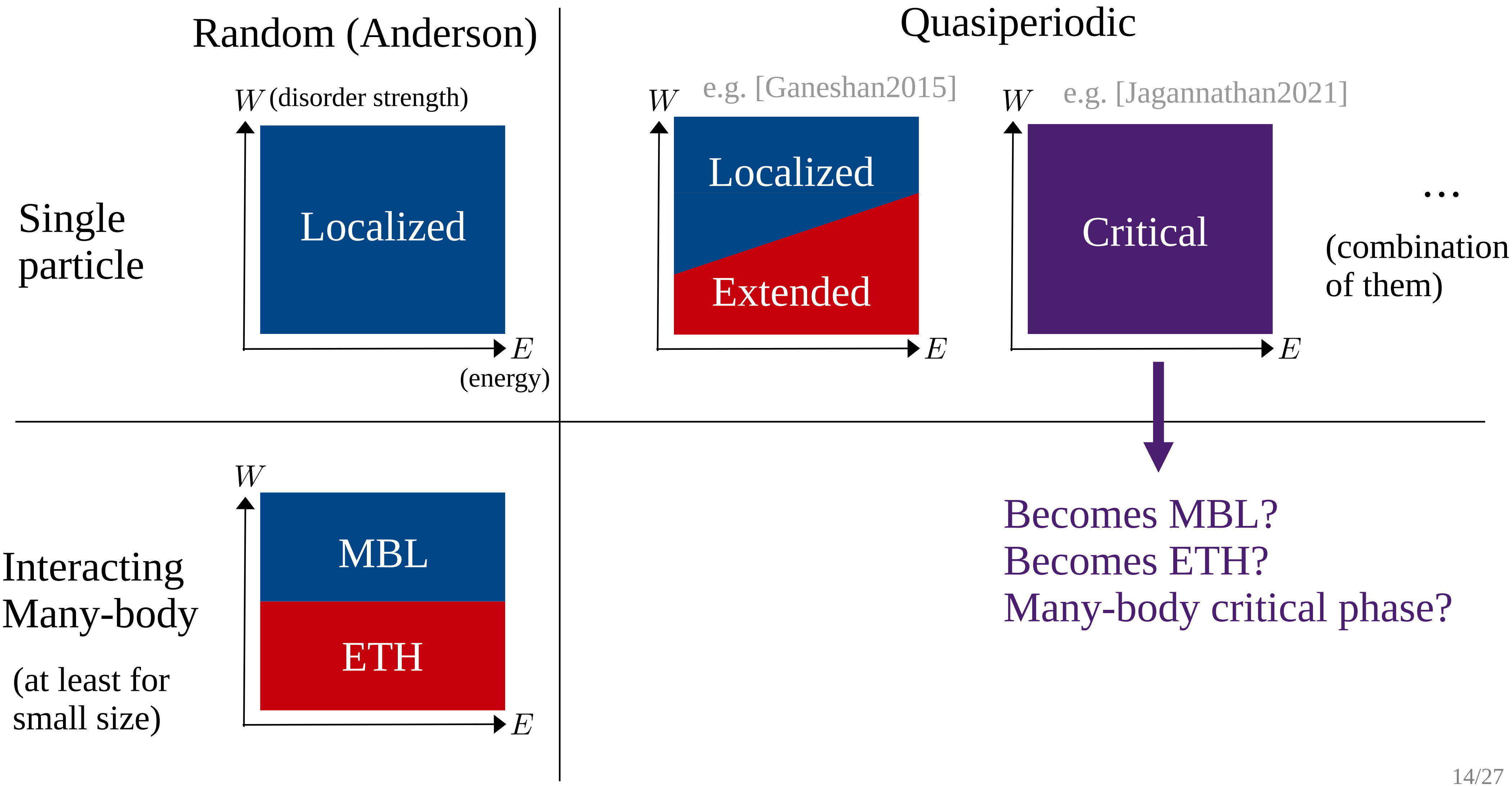
Interacting Many-body

(at least for small size)



Becomes MBL?
Becomes ETH?

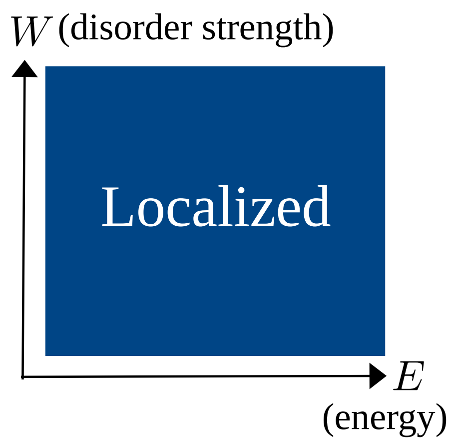
What’s special about 1D quasiperiodic systems



What’s special about 1D quasiperiodic systems

Random (Anderson)

Single particle



Interacting Many-body

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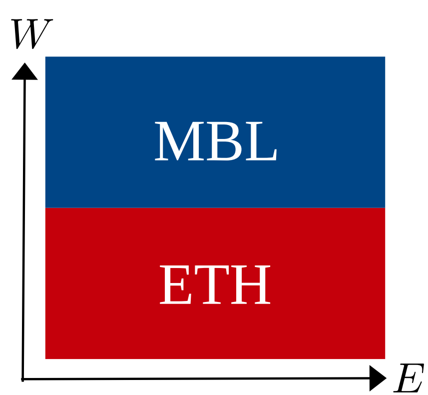
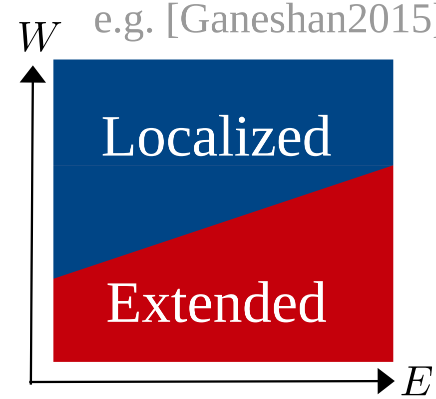
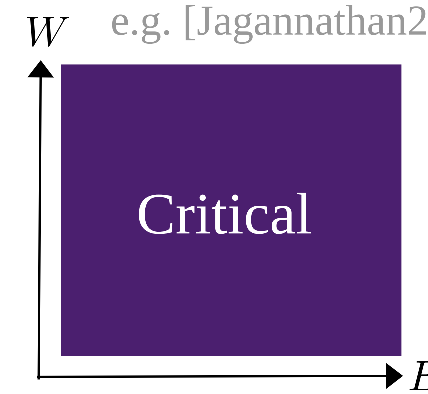


Quasiperiodic

e.g. [Ganeshan2015]



e.g. [Jagannathan2021]



…

(combination of them)

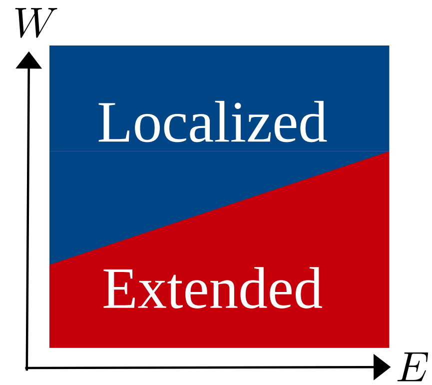
Becomes MBL?
Becomes ETH?
Many-body critical phase?

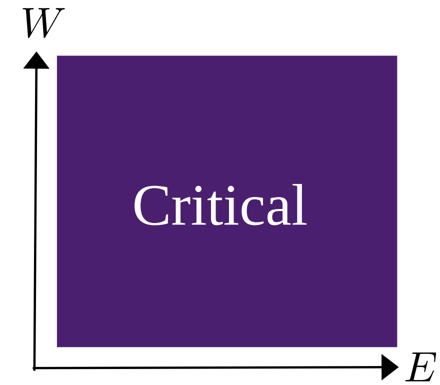
Existing literature are contradictory

e.g. [Mace2019,Varma2019,Wang2021]

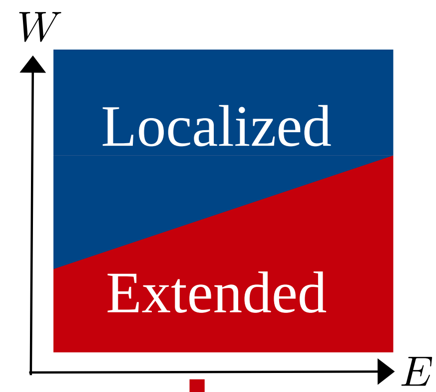
And often based on small size numerics

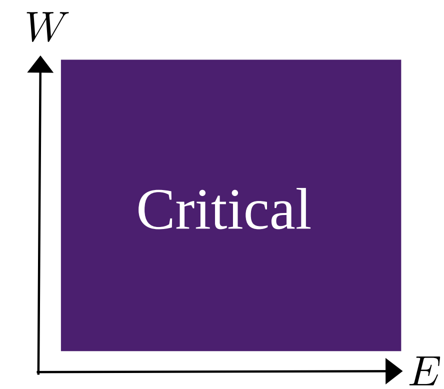
Single particle

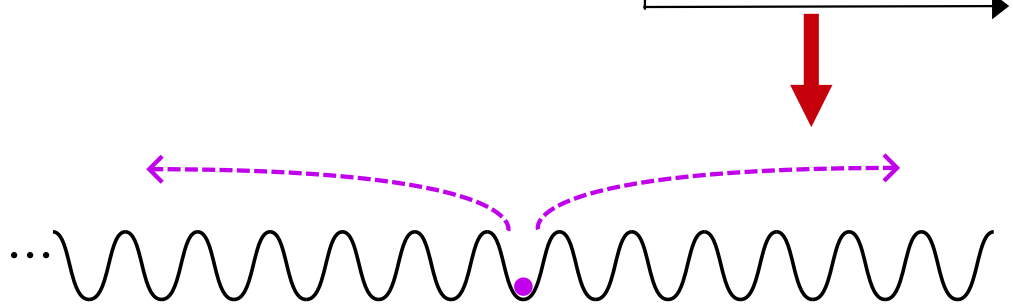




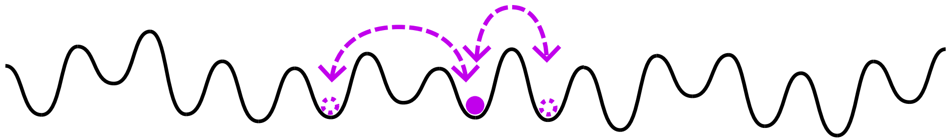
Single
particle





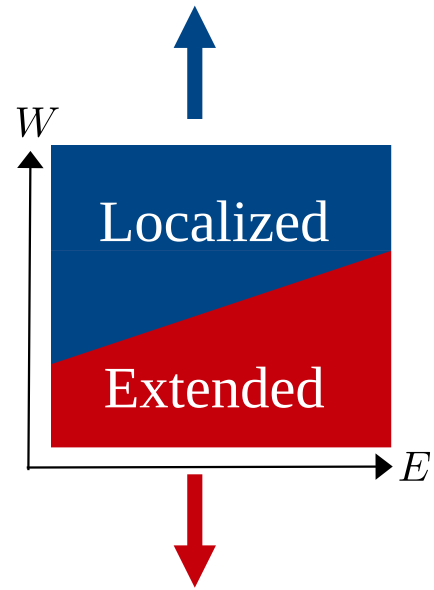


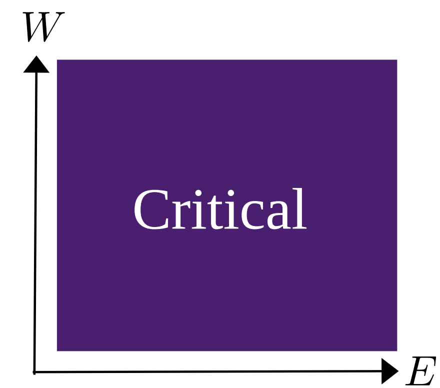
Travels further and further away
and never comes back

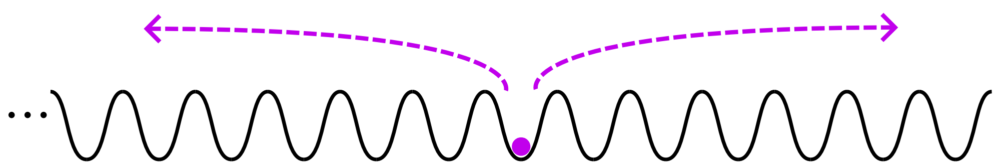


Comes back regularly
and does not go arbitrarily far away

Single
particle







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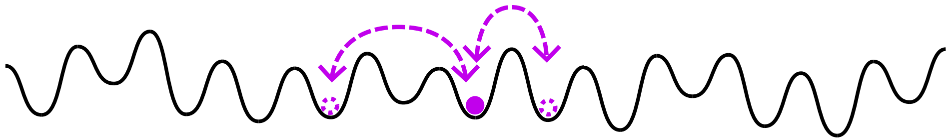

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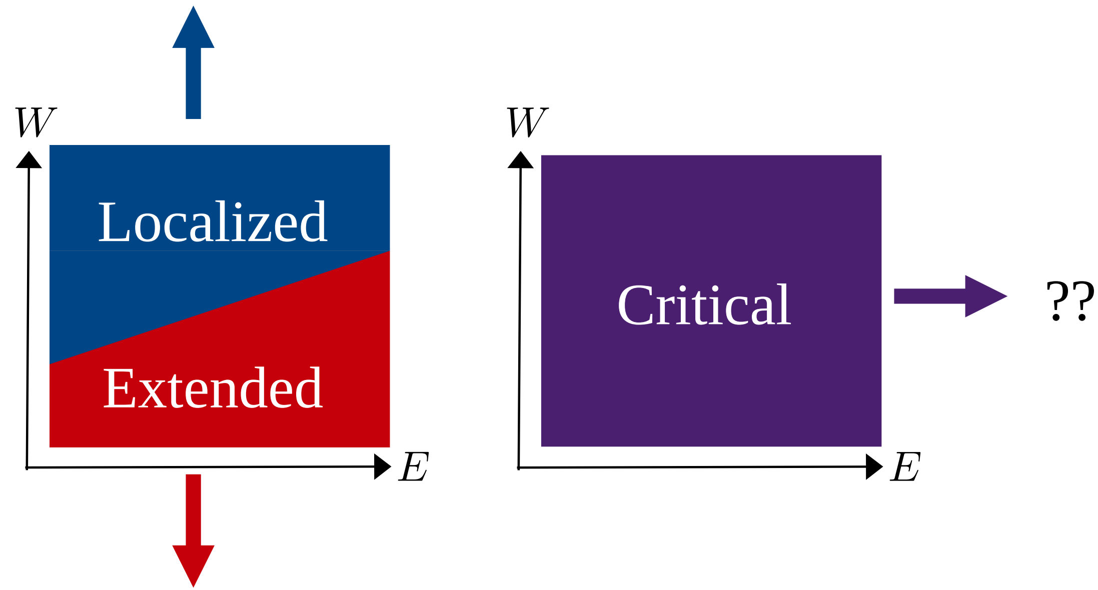

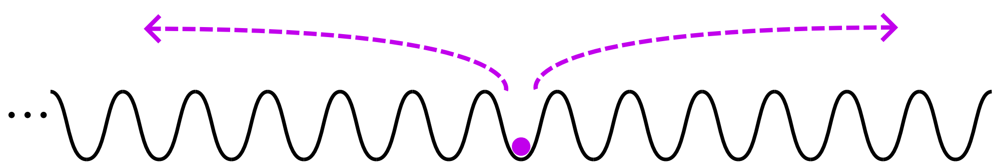

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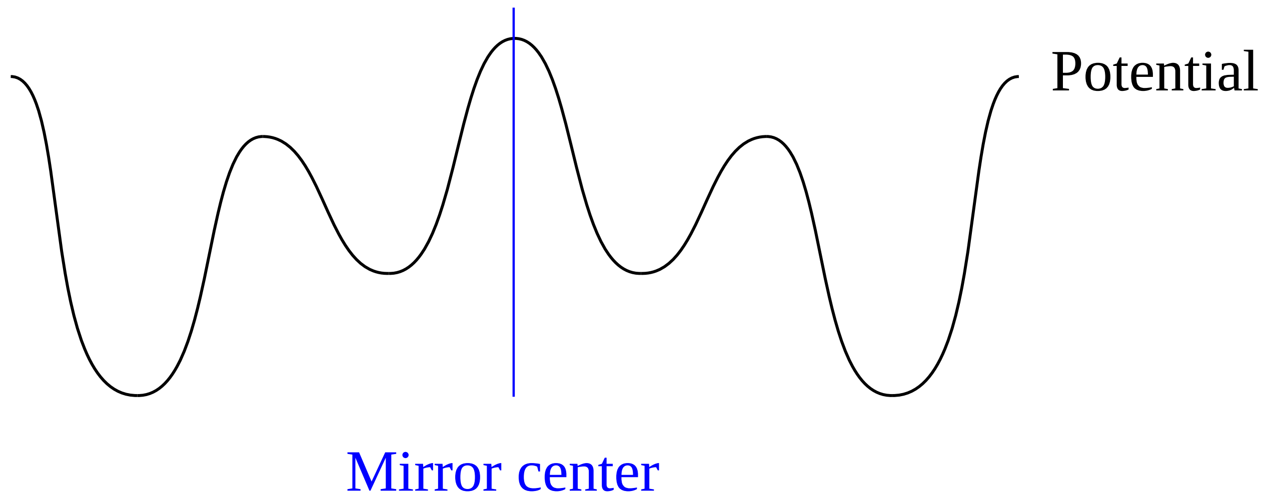
Hierarchical mirror structure (HMS)



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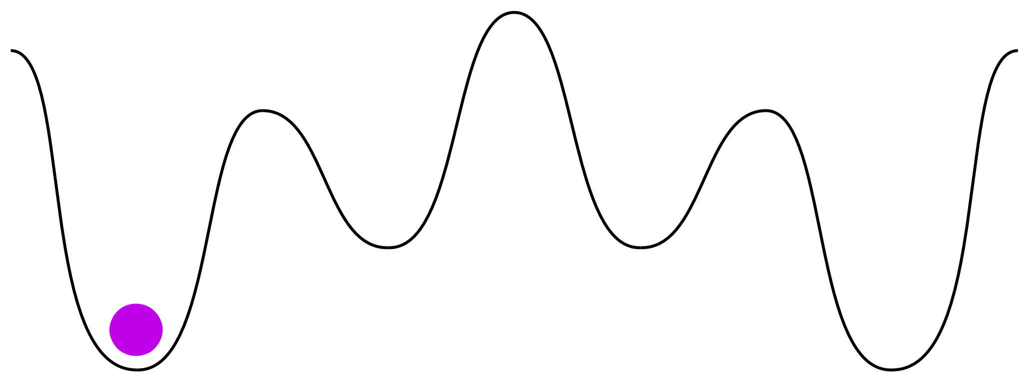

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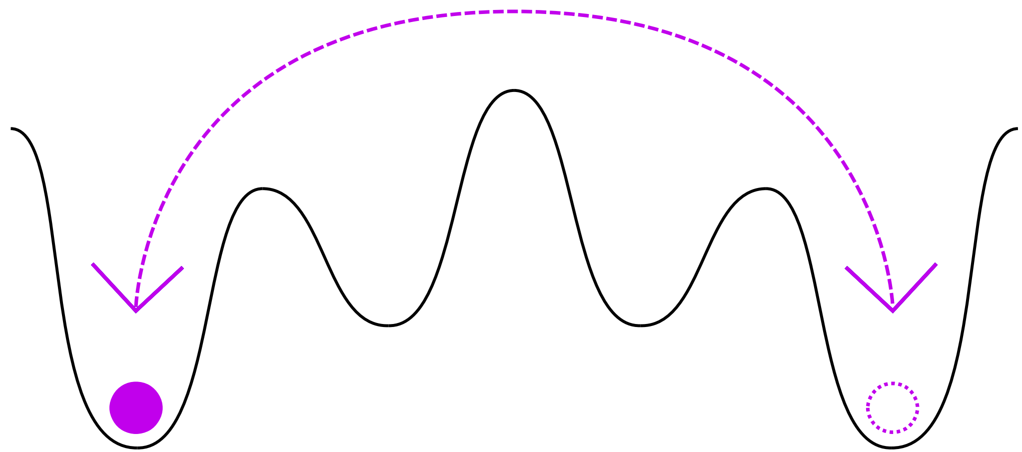

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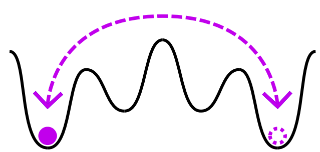

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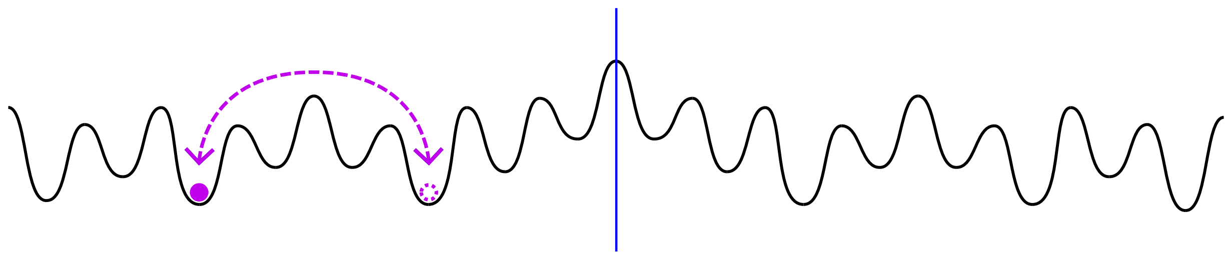

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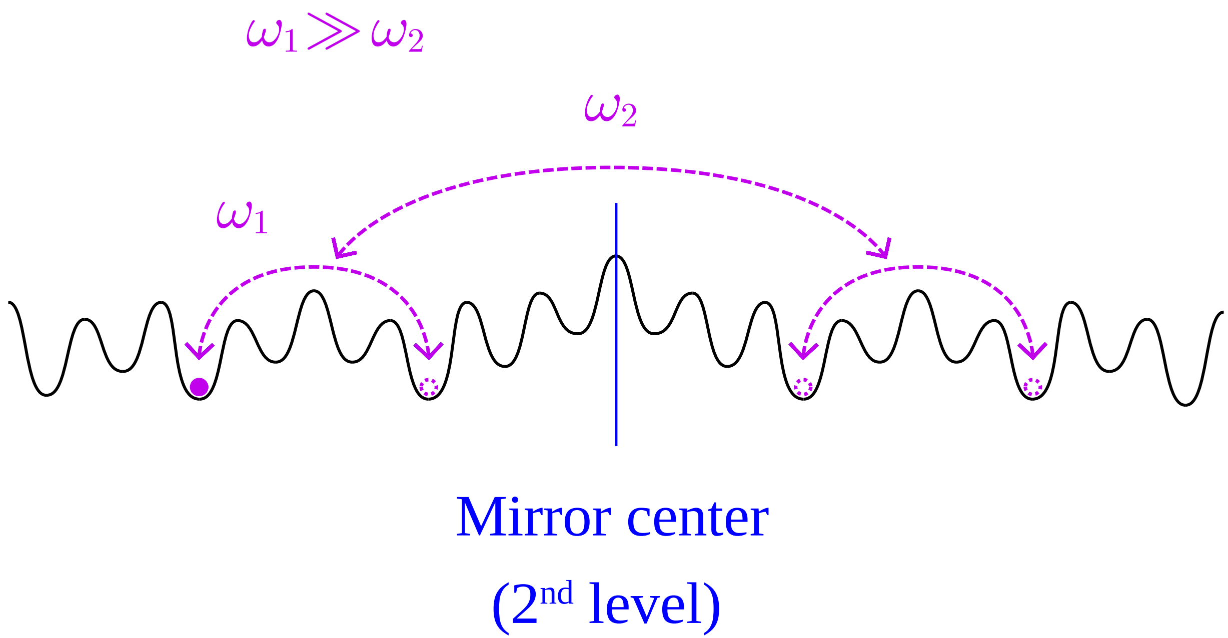
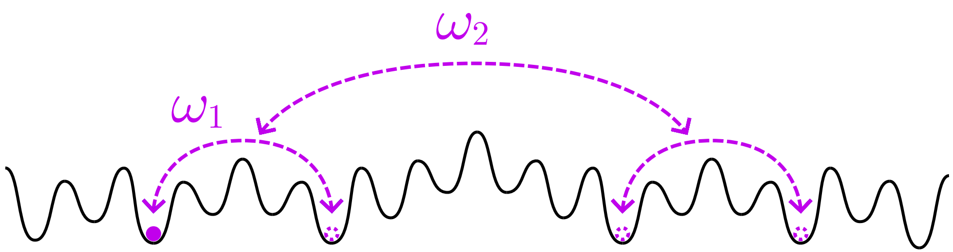


Hierarchical mirror structure (HMS)

$\omega_1 \gg \omega_2$



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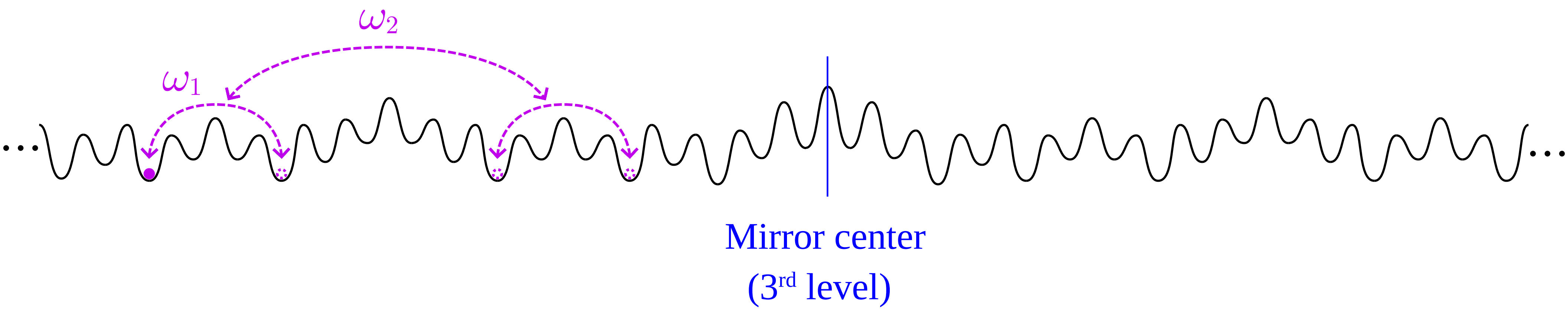

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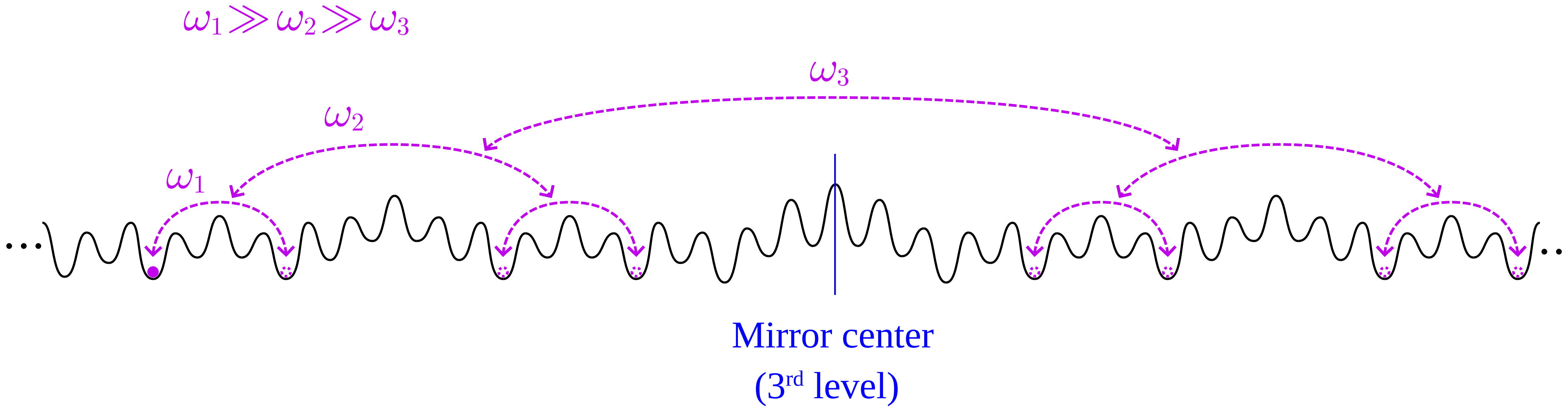
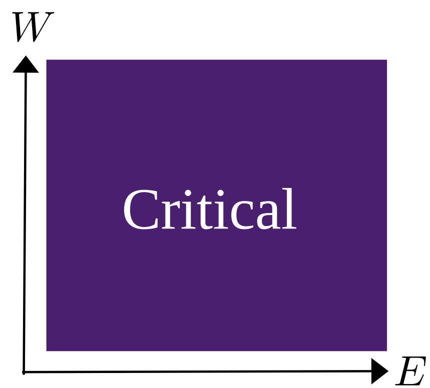
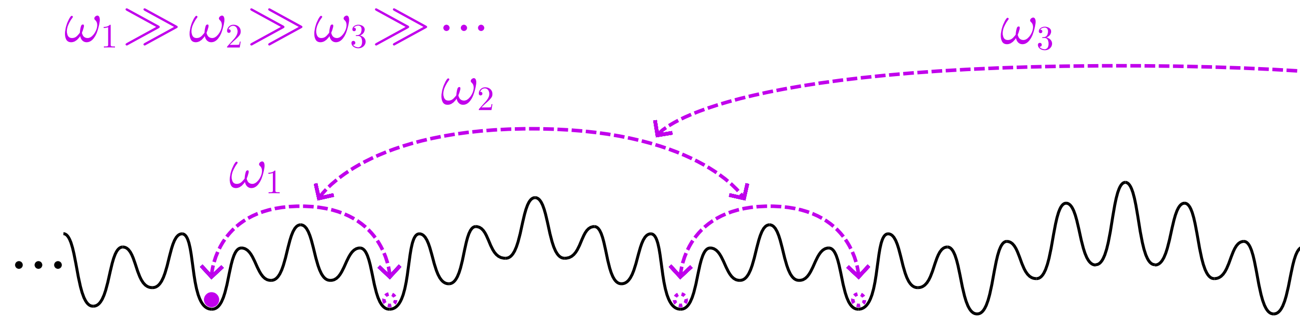


Critical phases in many quasiperiodic systems has HMS
- Fibonacci quasicrystal [Jagannathan2021]
- Extended Aubry-Andre model [Avila2017]

Single particle





The particle goes arbitrarily far away,
but also comes back after arbitrarily long time

Interacting Many-body

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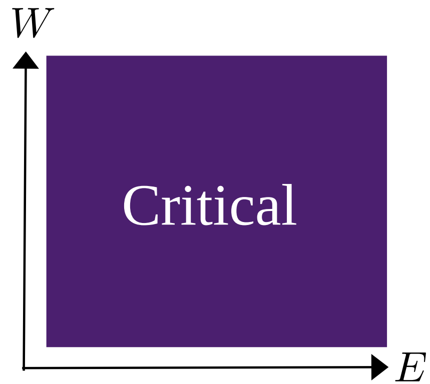

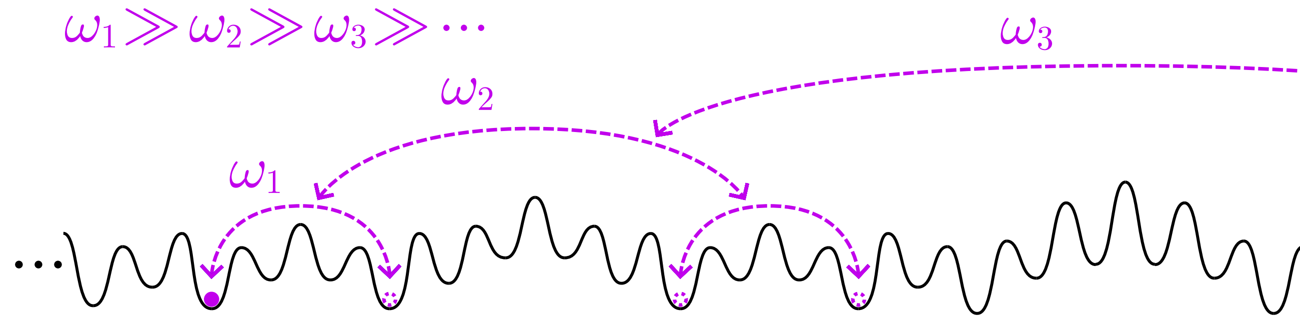

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Interacting Many-body

Becomes MBL?
Becomes ETH?
Many-body critical phase?

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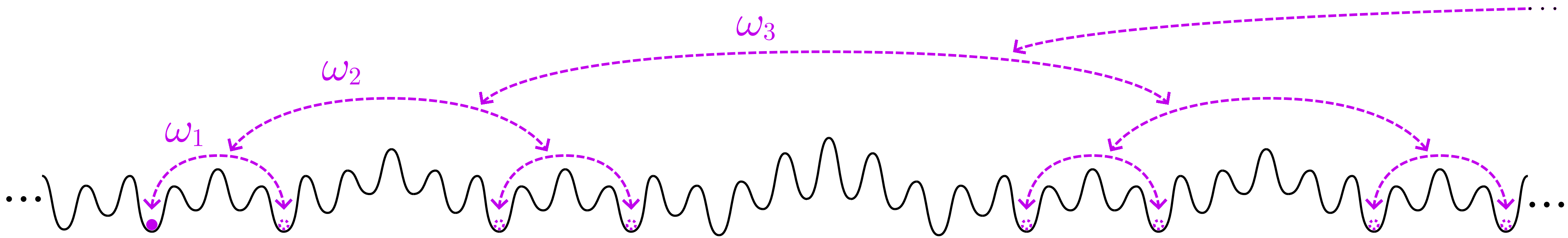
Restriction to the first level

$\omega_1 \gg \omega_2 \gg \omega_3 \gg \cdots$



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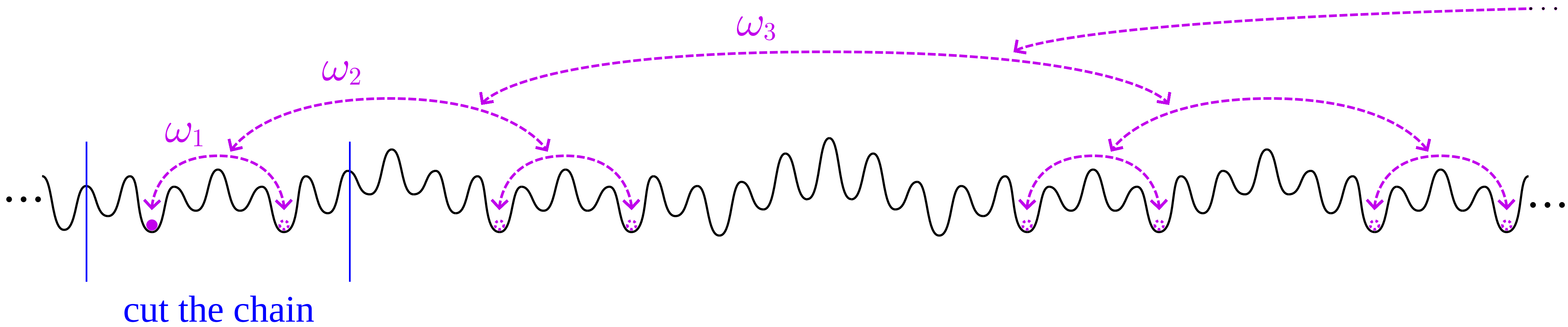

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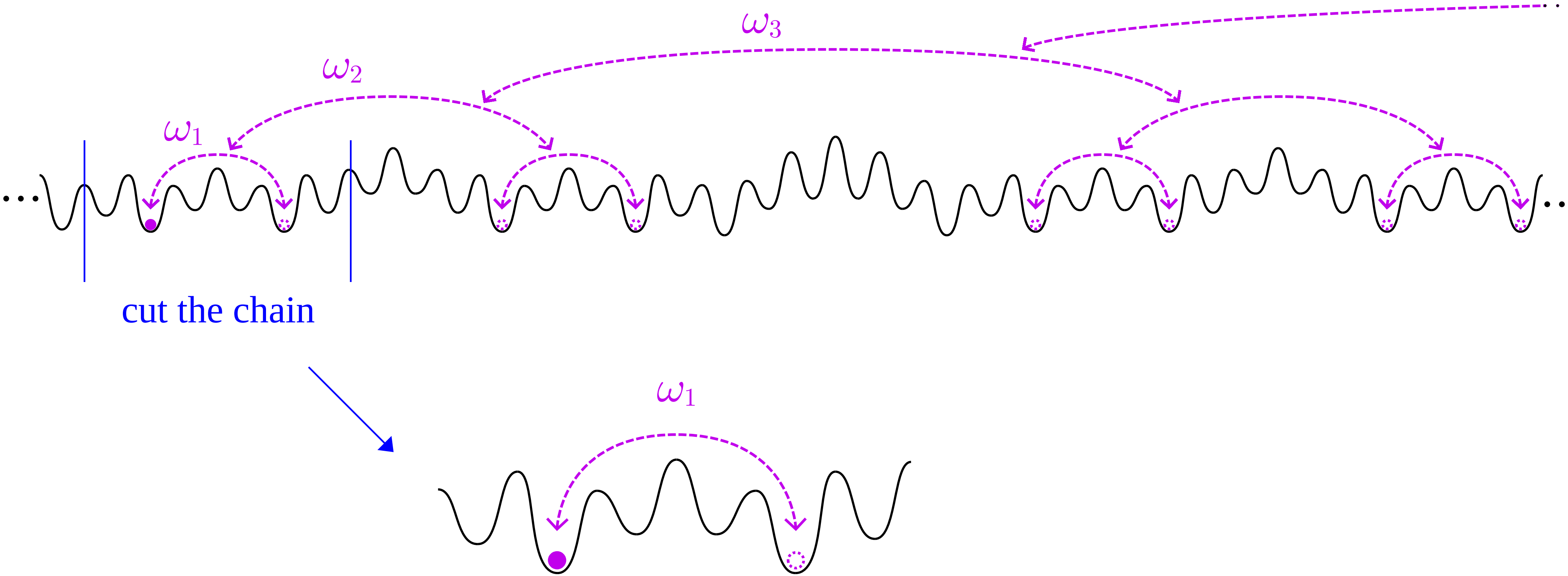
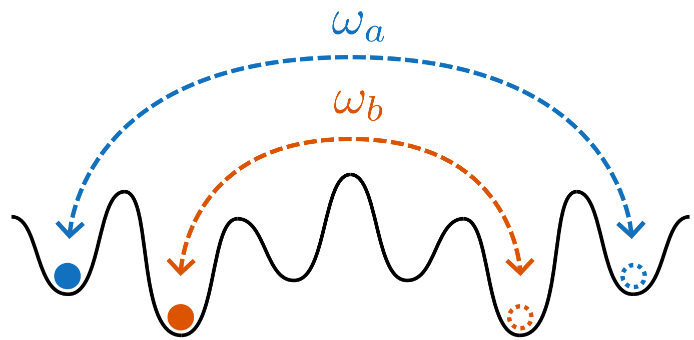


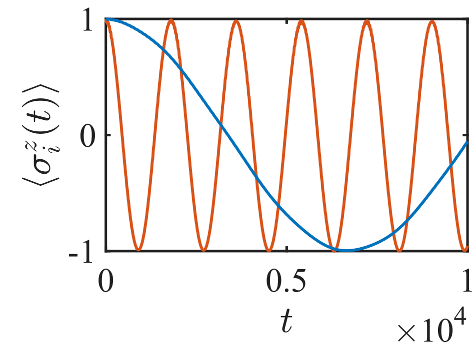
Resonance (oscillation) in the first level

Effects of interaction in the first level

Li, YTT, Das Sarma PRB (2026)

Non-interacting:





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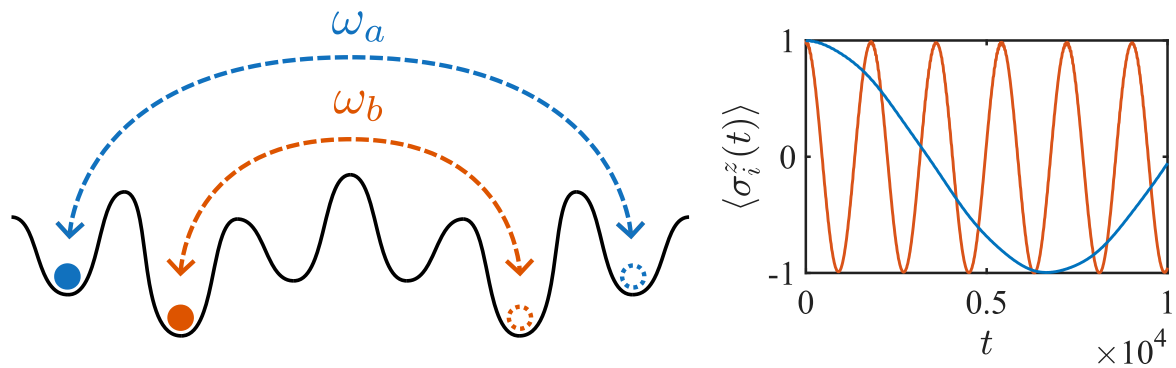
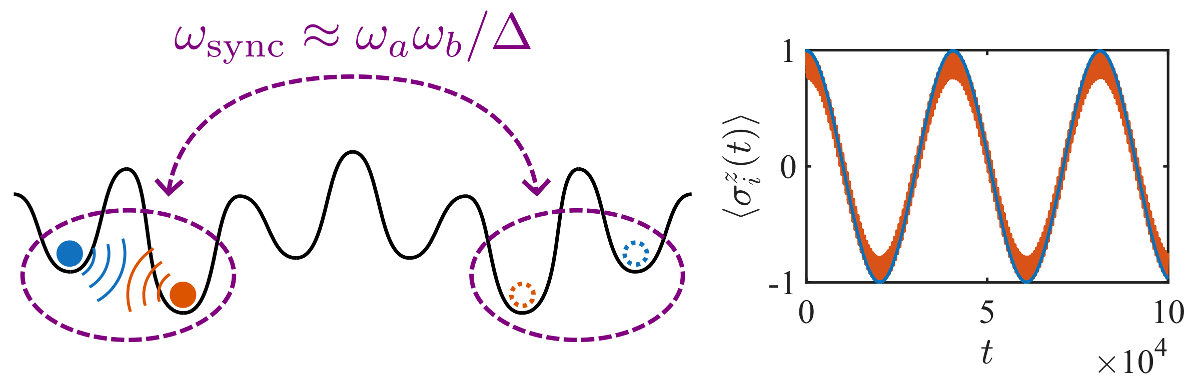


Interacting:



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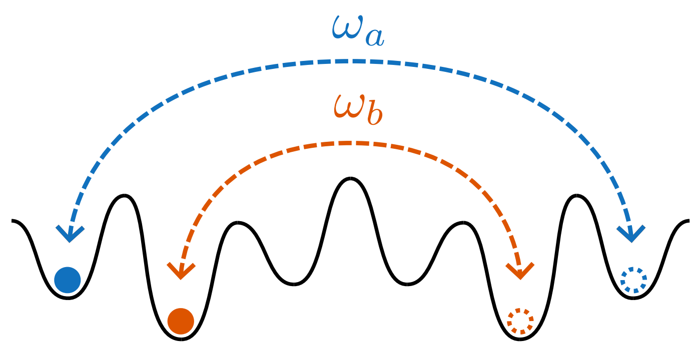
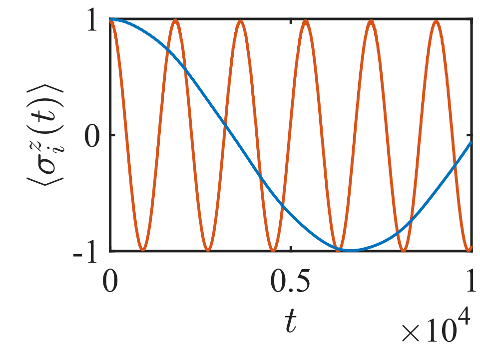
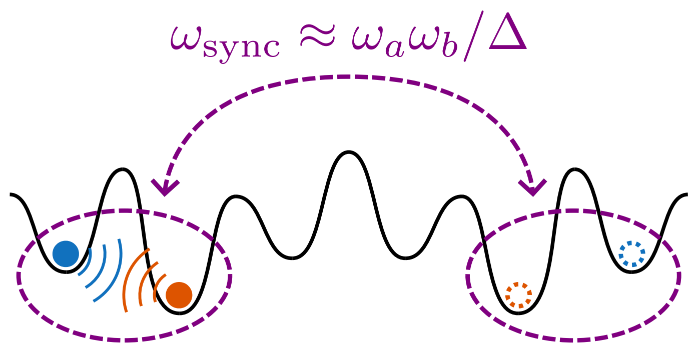
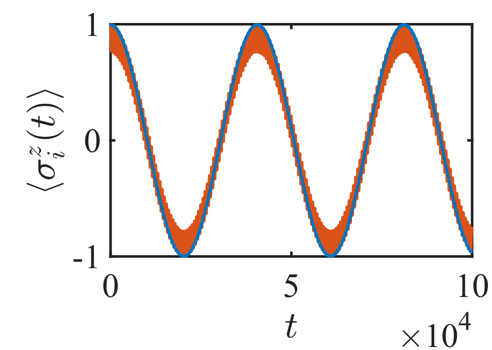




Unsynchronized

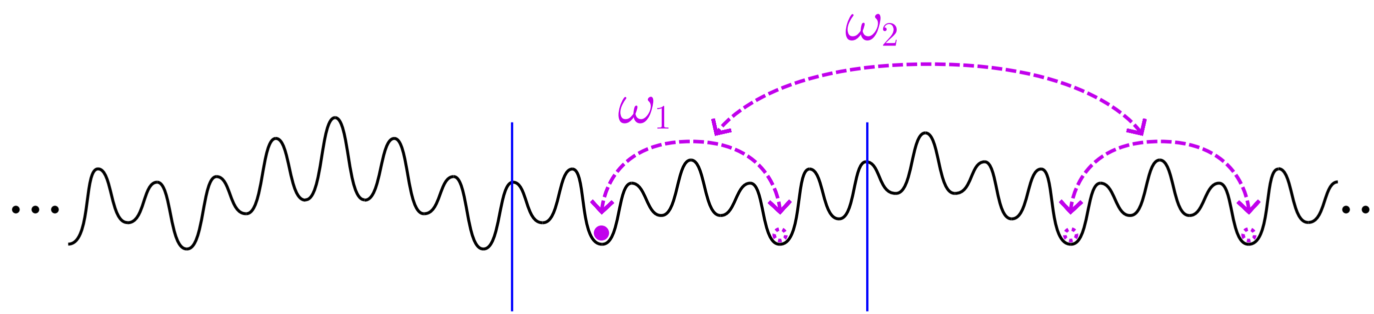
Interacting:





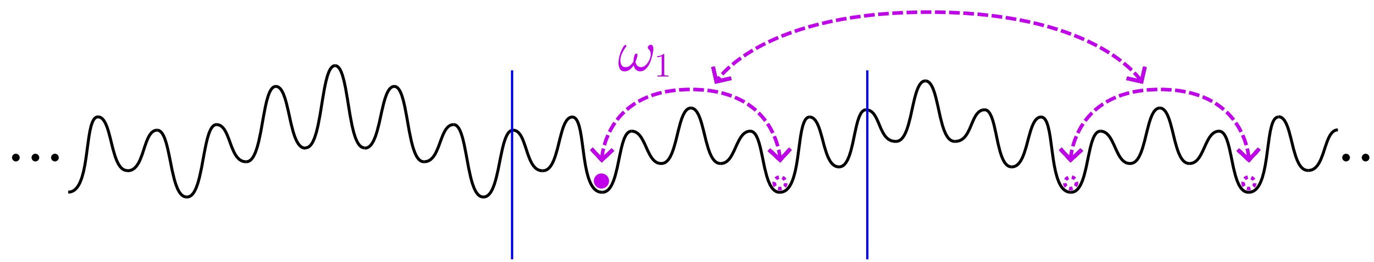
Synchronized

Symmetry breaking

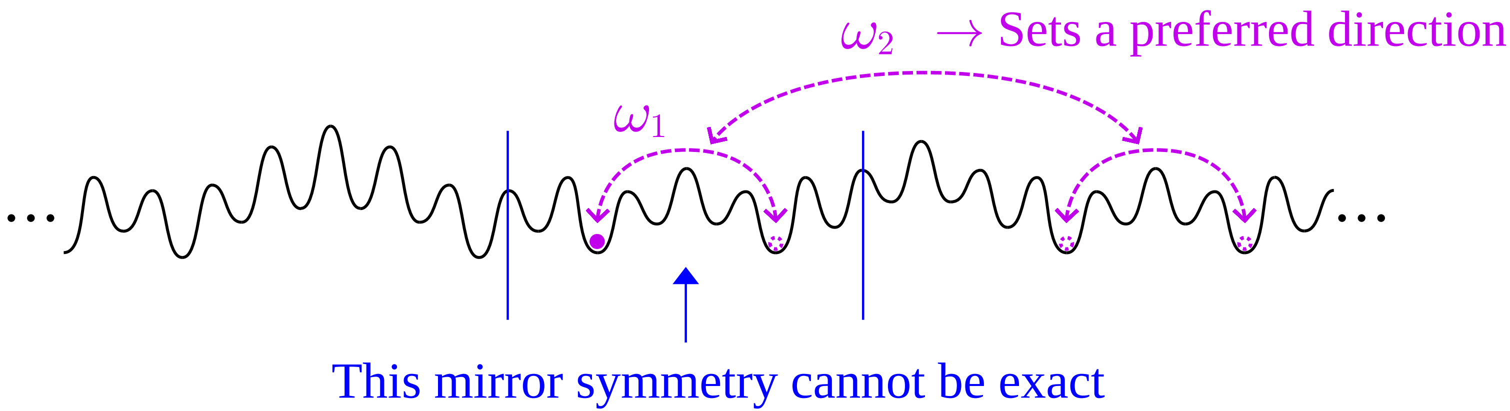


Symmetry breaking

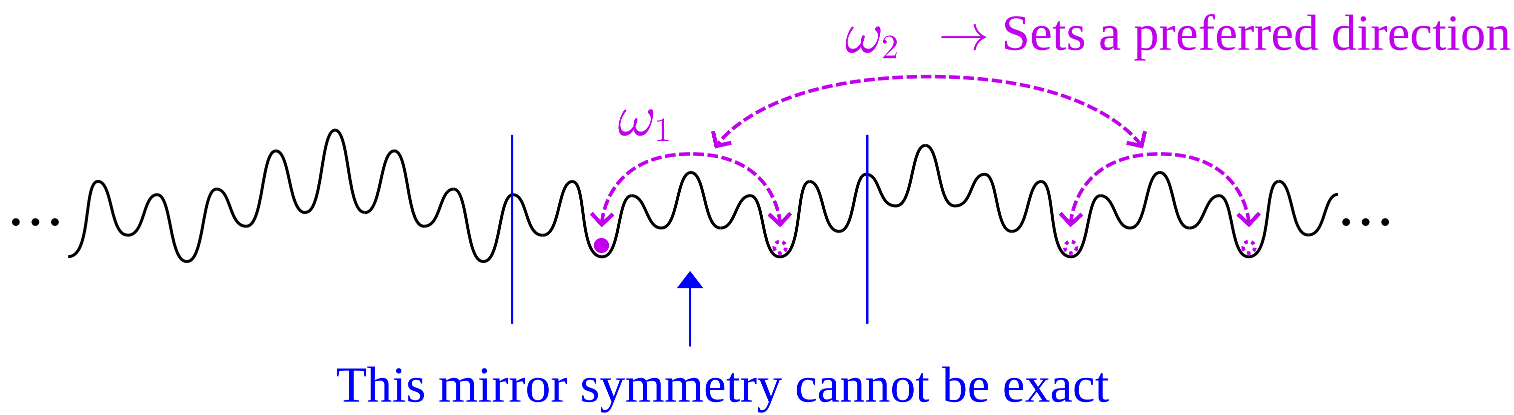
$\omega_2 \quad \rightarrow$ Sets a preferred direction

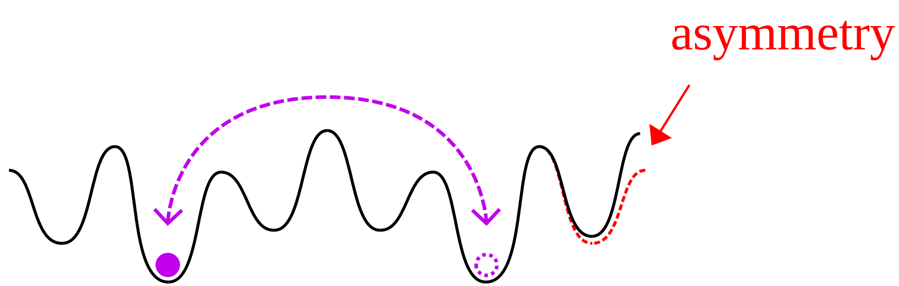


Symmetry breaking



Symmetry breaking





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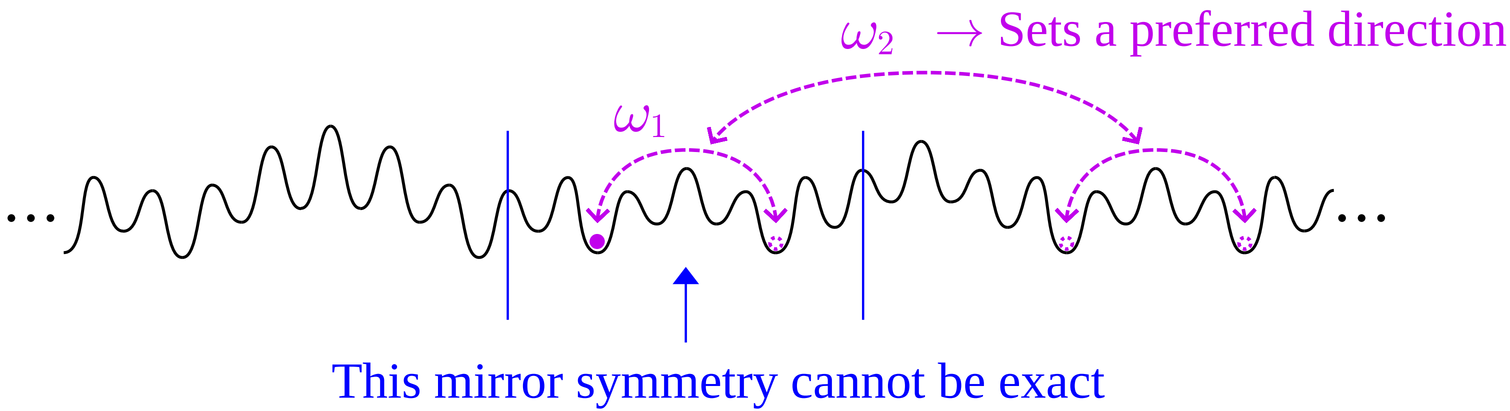

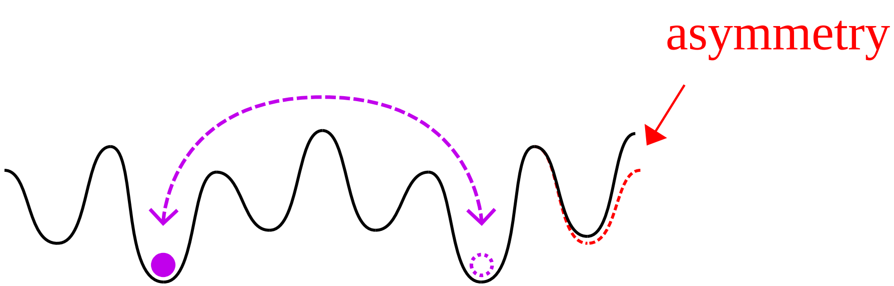

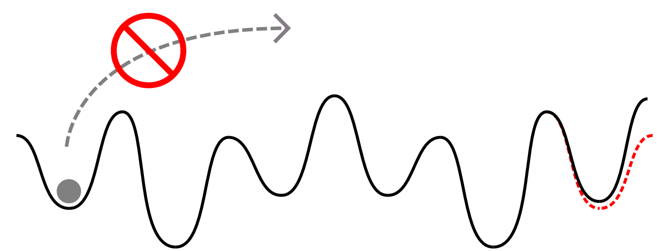

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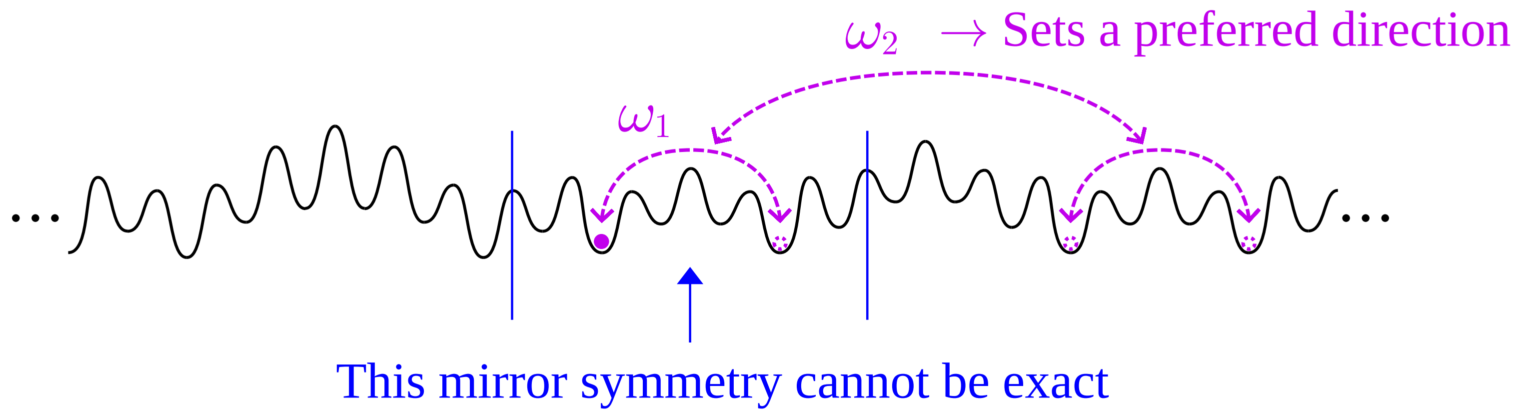
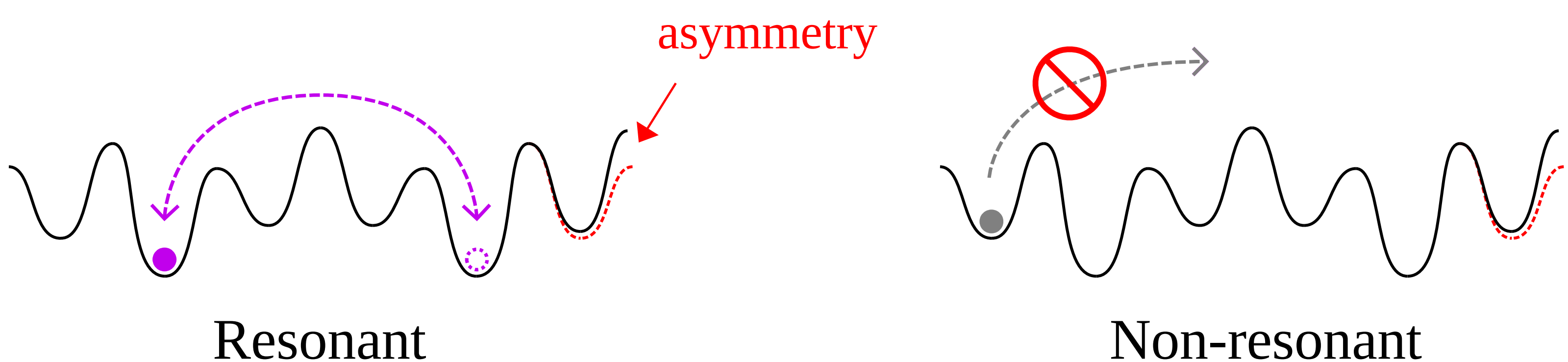
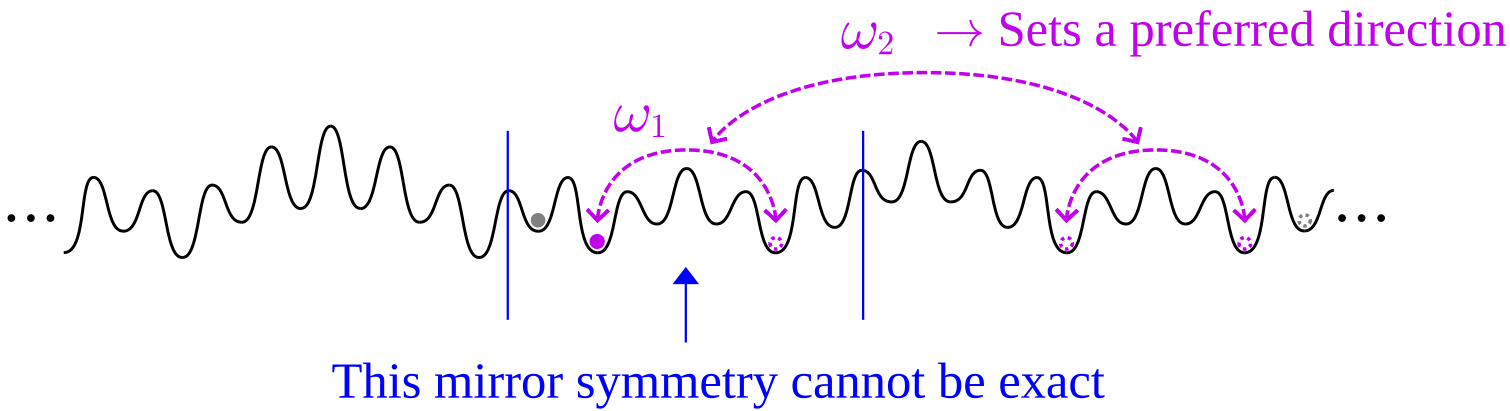
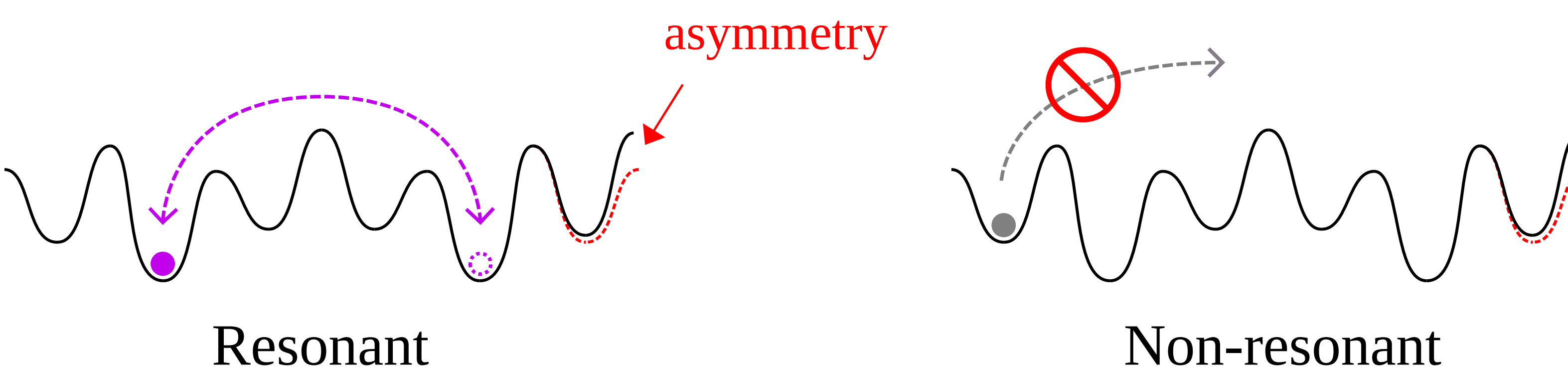




A particle too far away from the mirror center becomes *non-resonant*

Symmetry breaking





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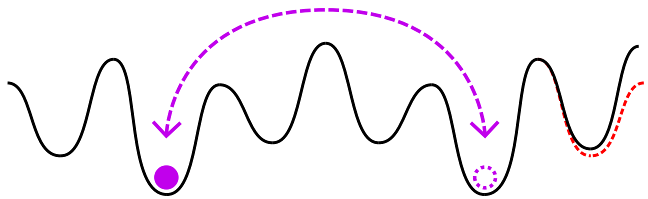
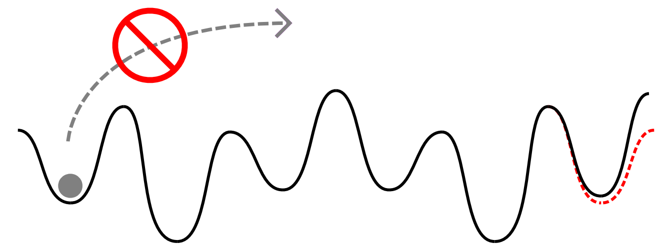
Interplay between interaction and symmetry breaking



Resonant



Non-resonant

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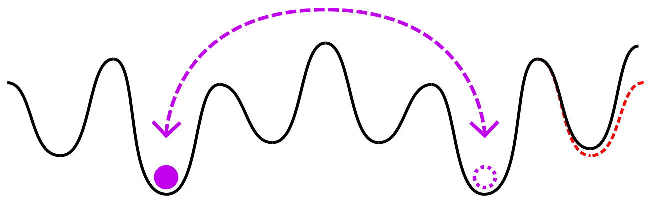

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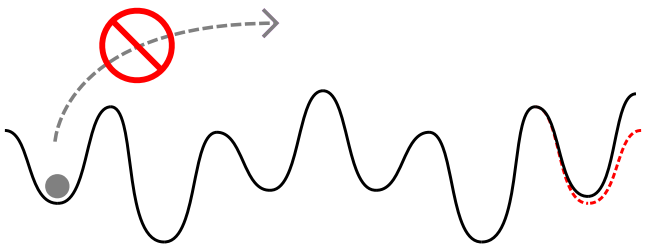

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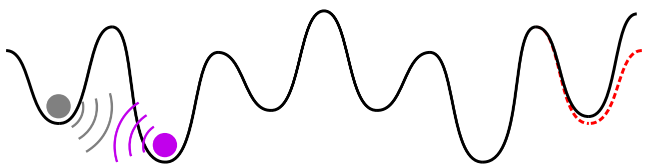

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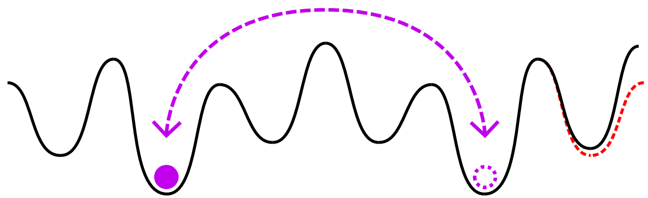

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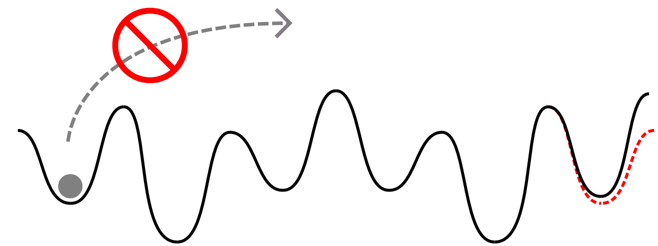

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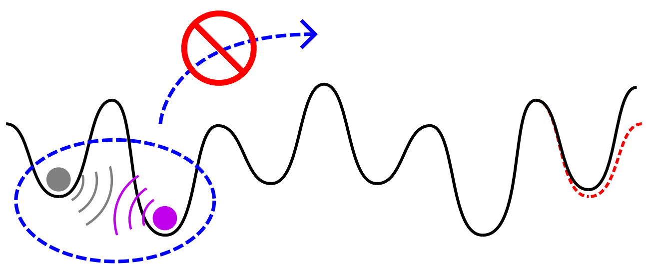

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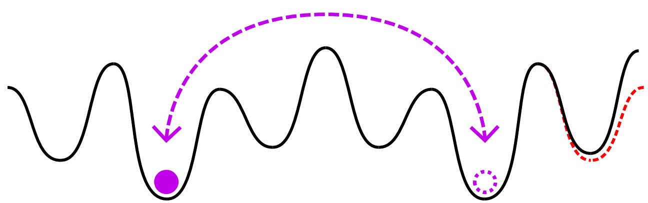

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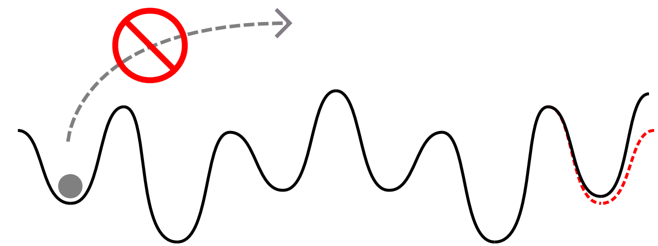

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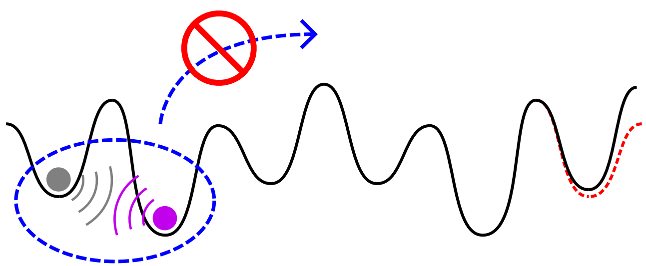

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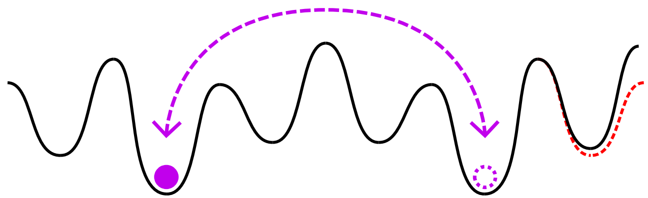

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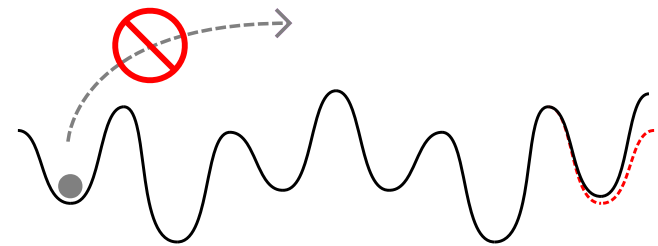

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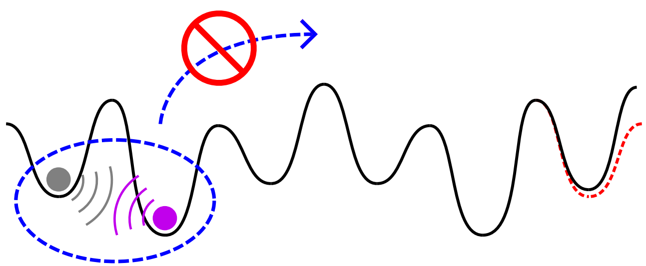

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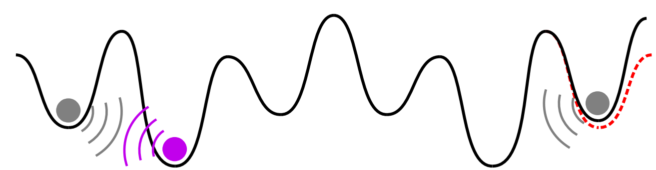

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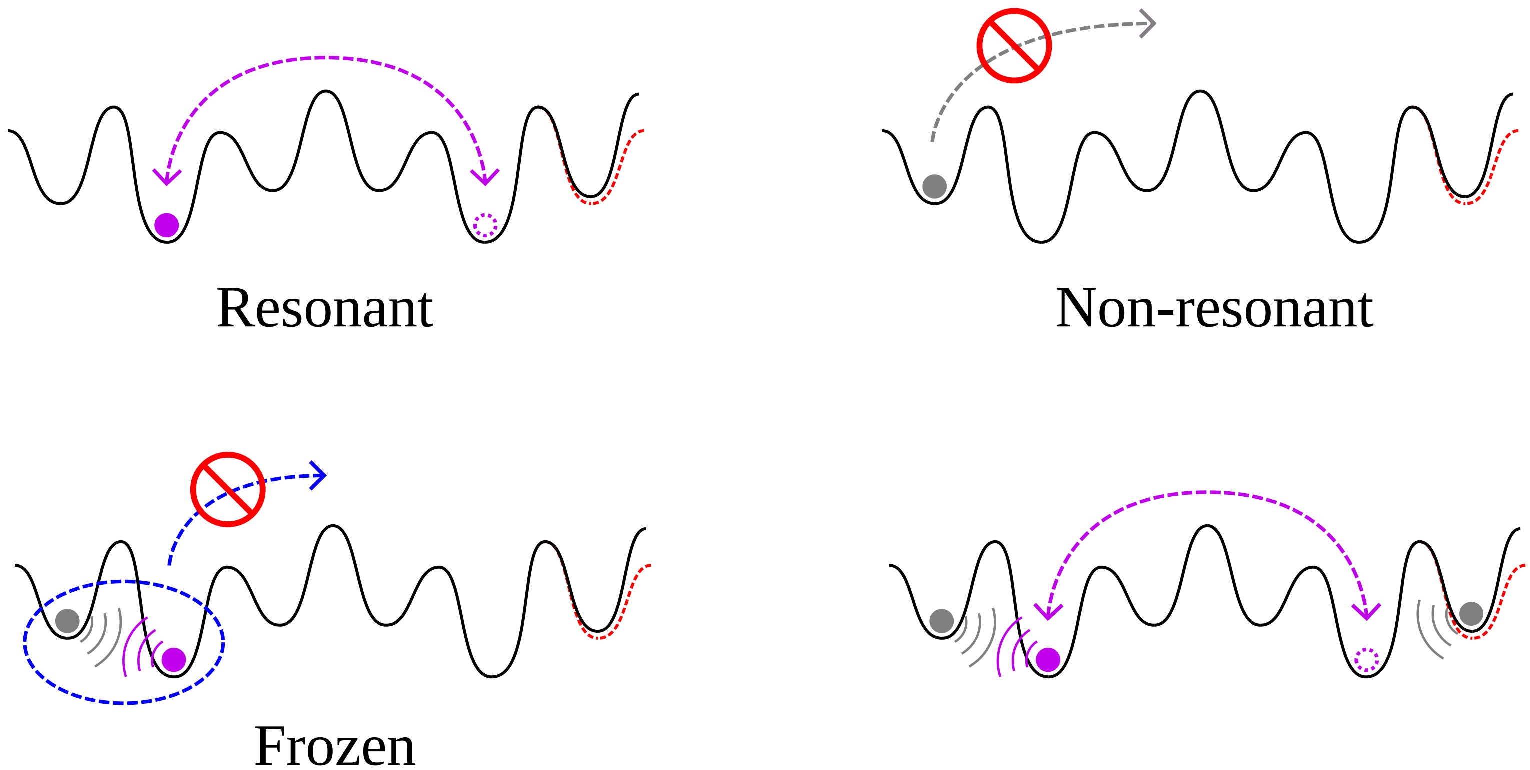

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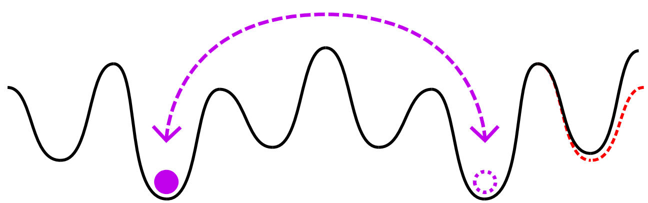

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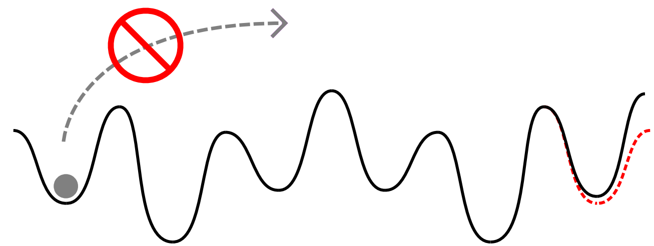

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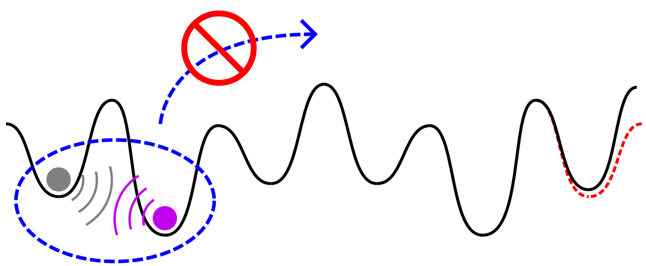

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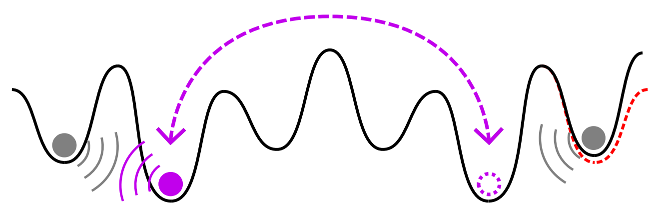
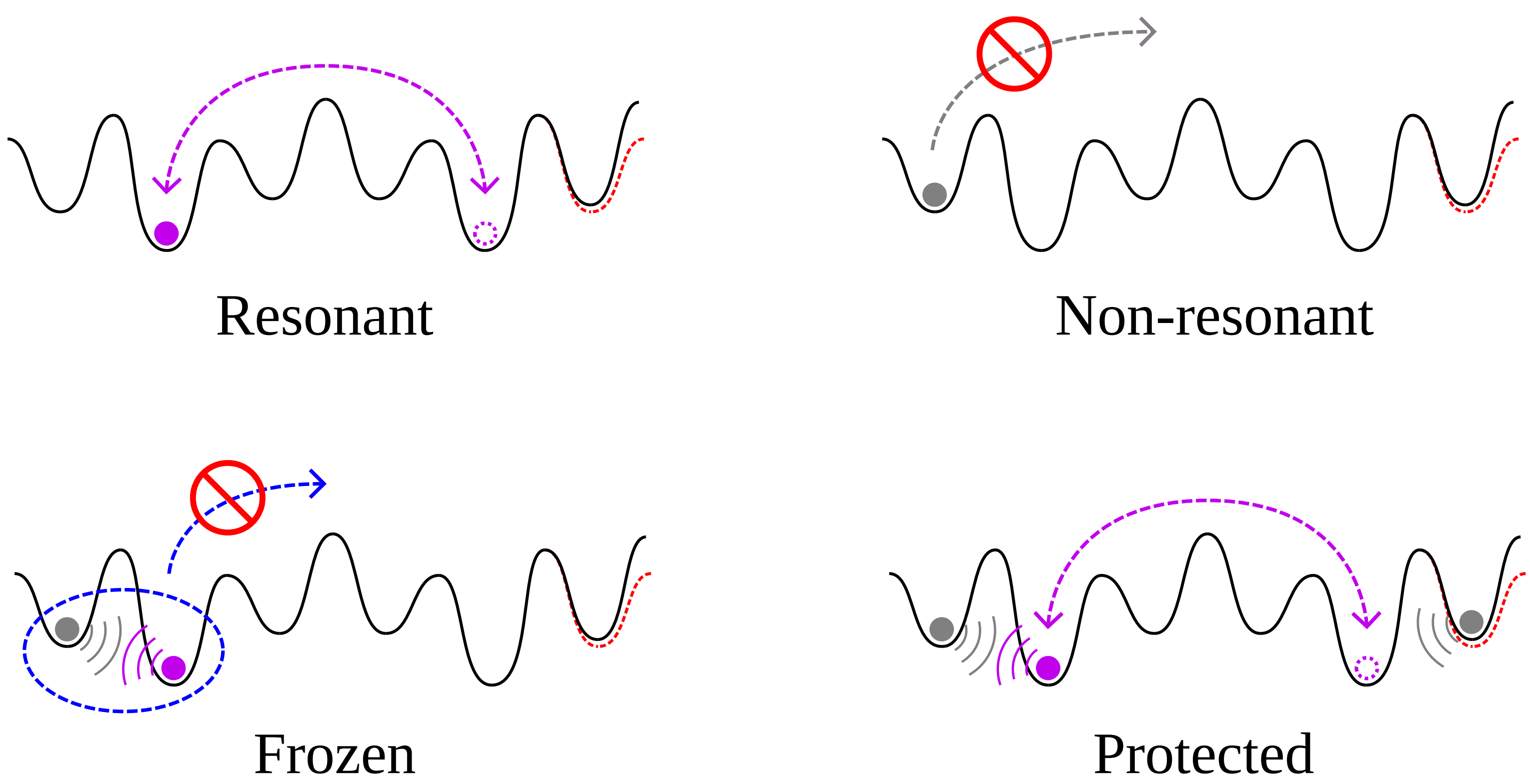


Protected

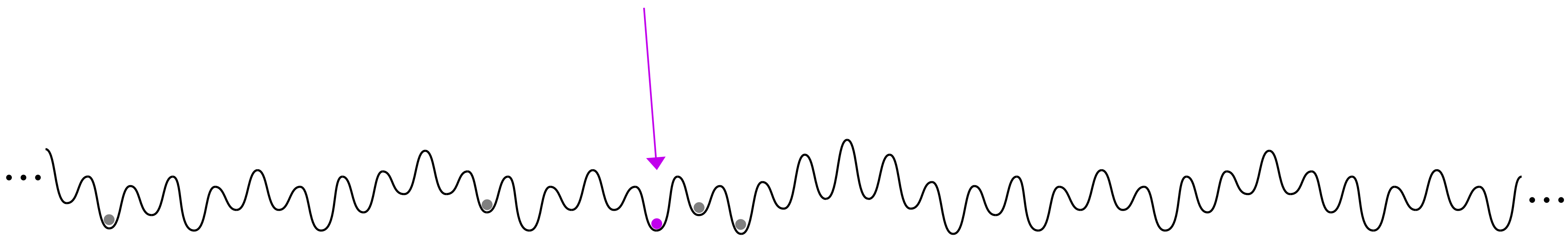
Interplay between interaction and symmetry breaking



Resonances are *protected* iff the configs of other particles are symmetric enough

Condition of delocalization in HMS with interactions

Given an initial many-body configuration, can the purple particle go arbitrarily far away?



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Given an initial many-body configuration, can the purple particle go arbitrarily far away?
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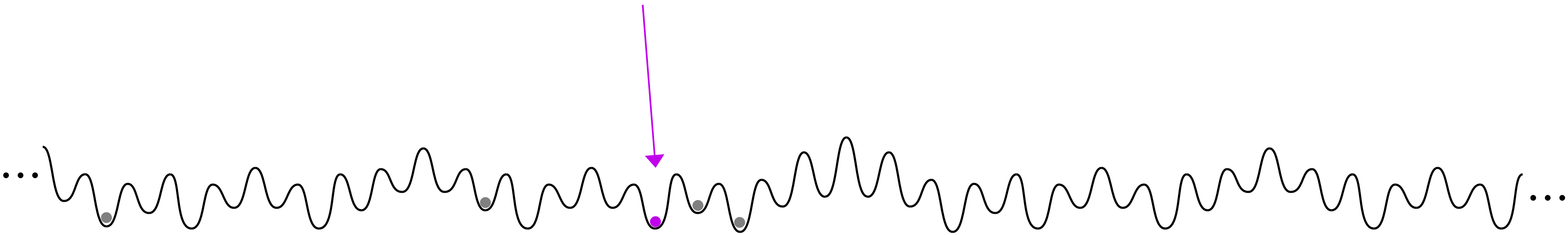

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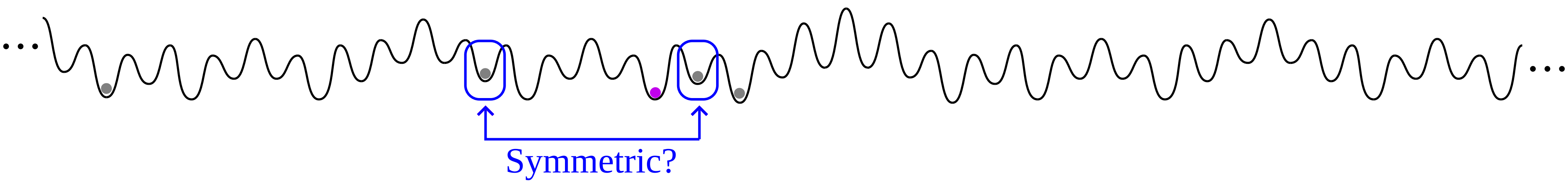

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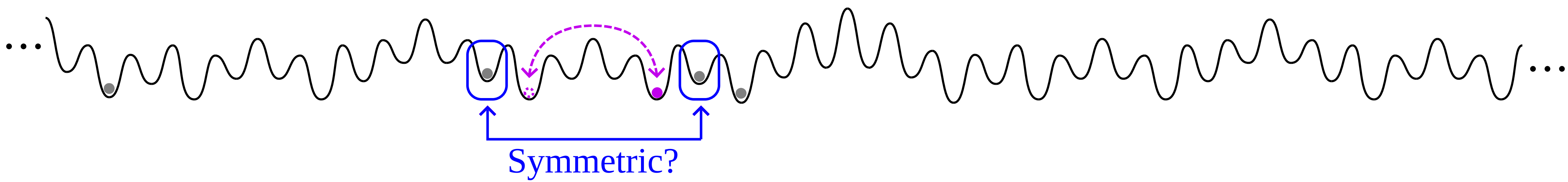

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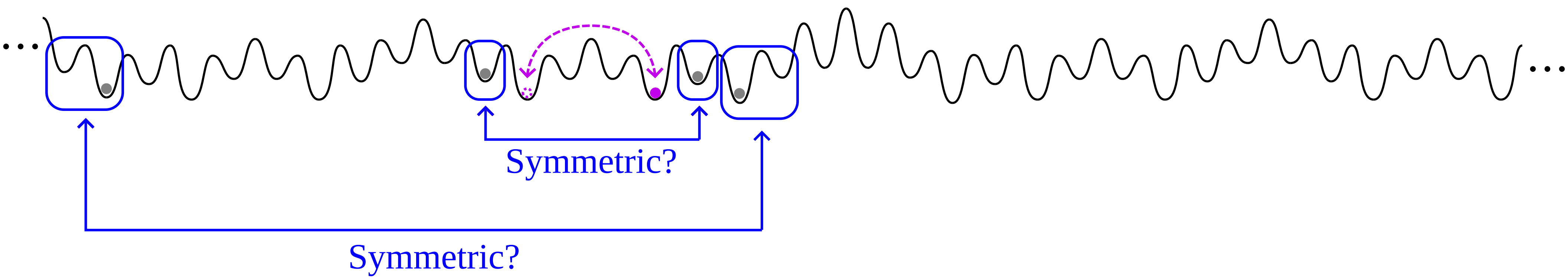

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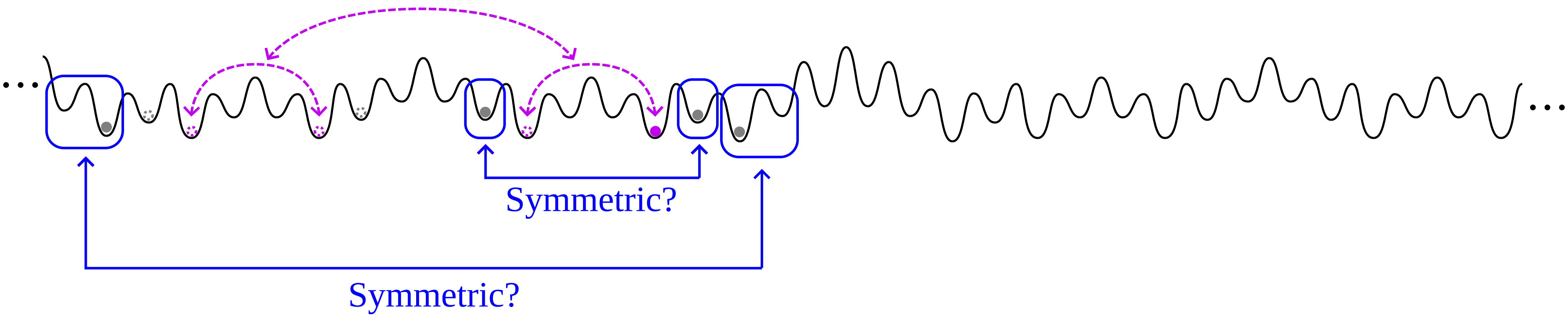

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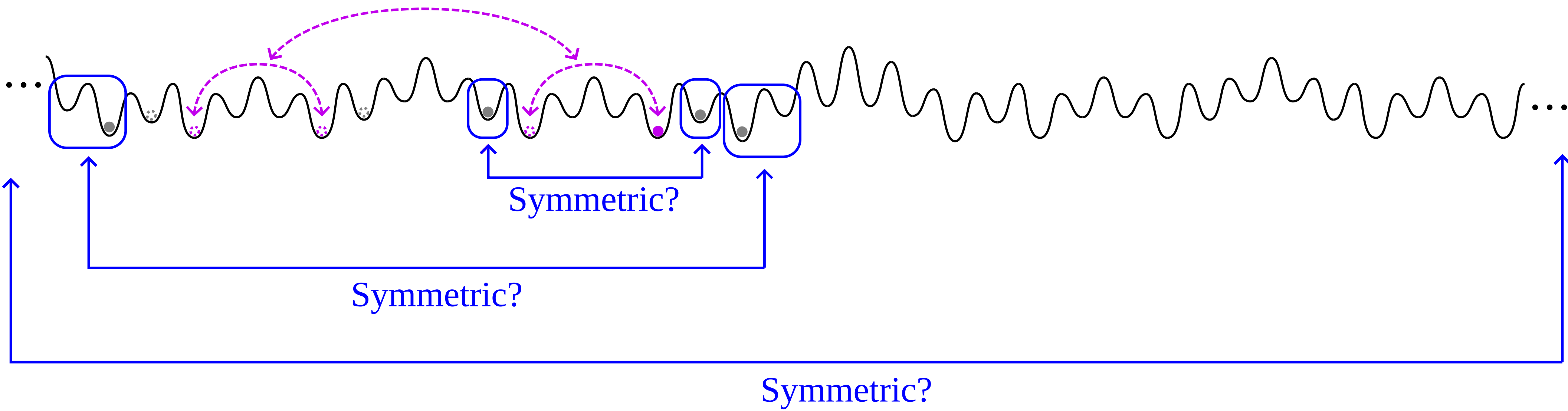

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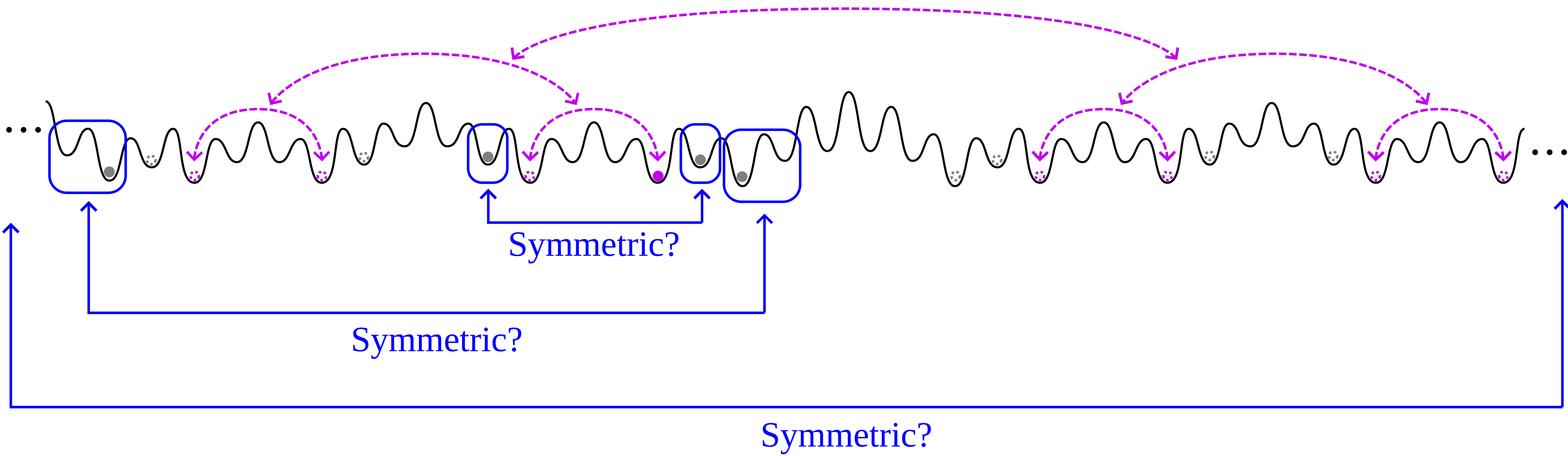

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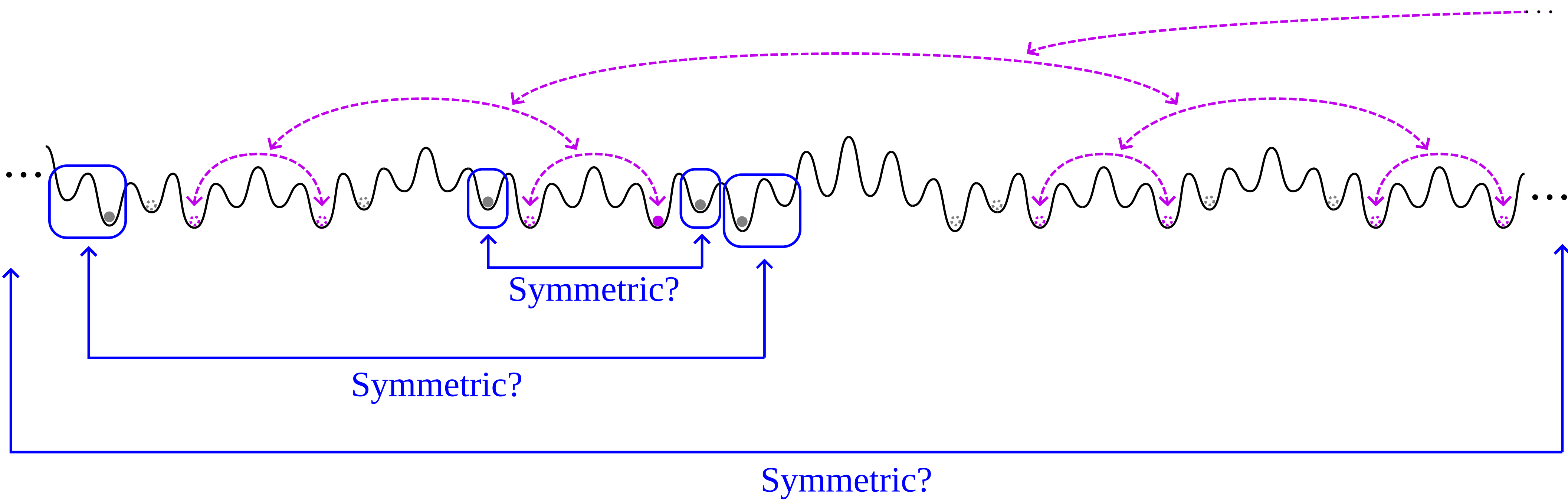

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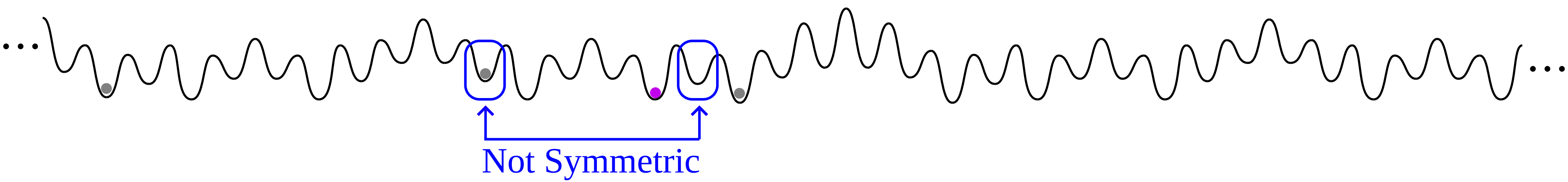

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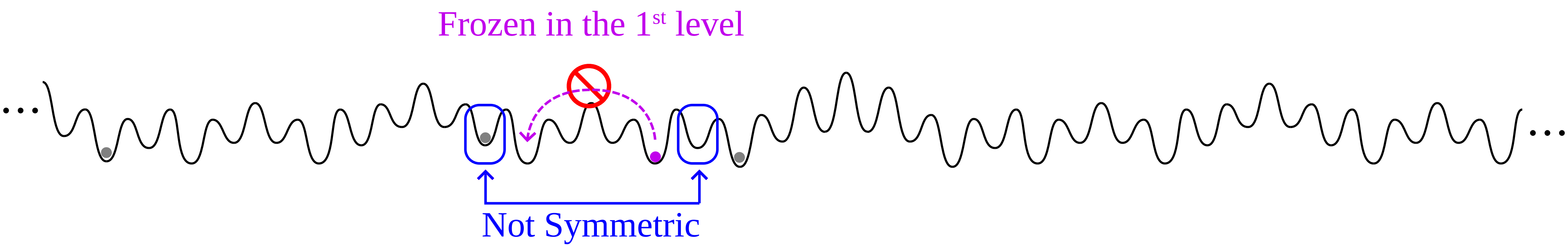

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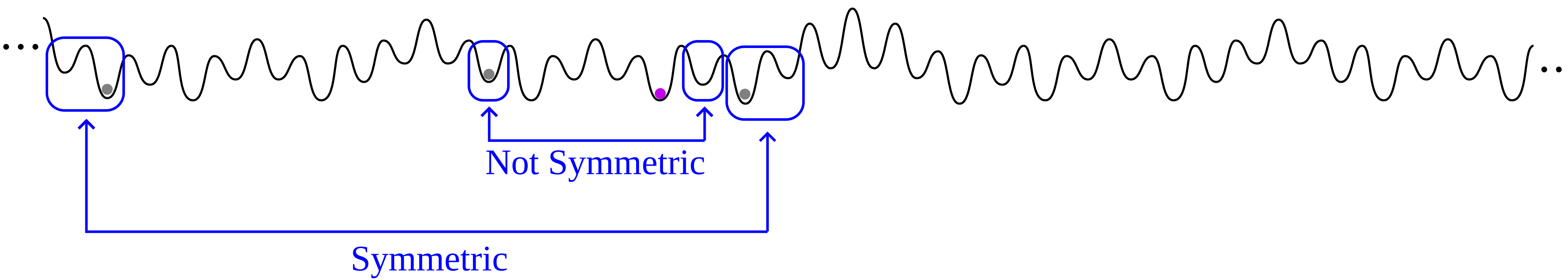

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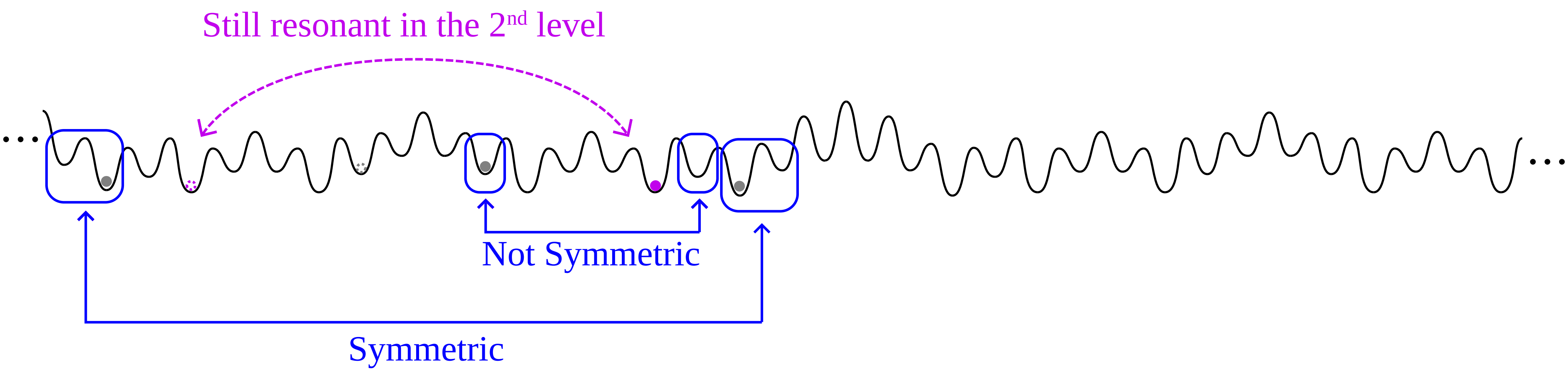

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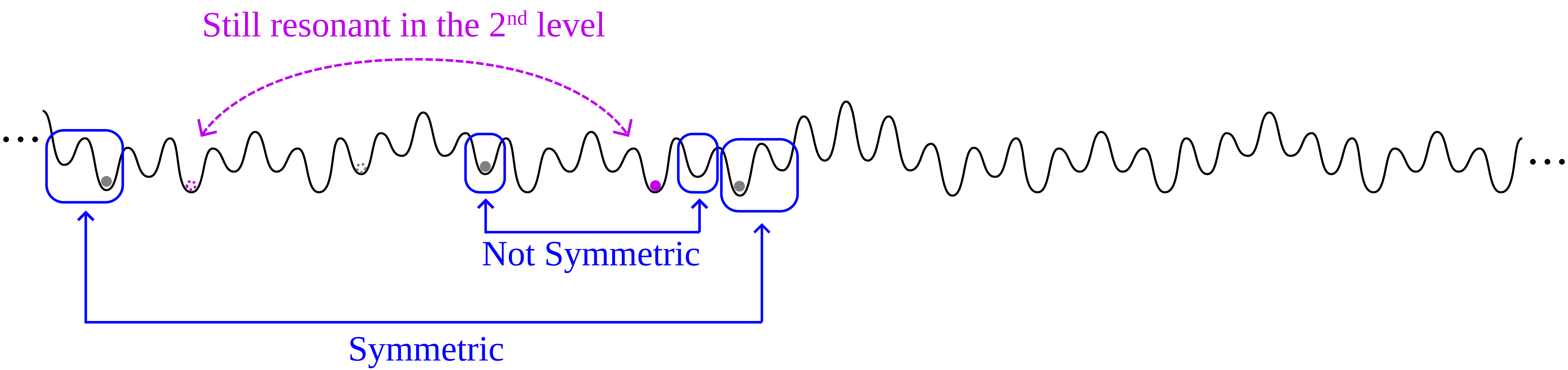

As long as there are resonances at level n for arbitrarily large n,

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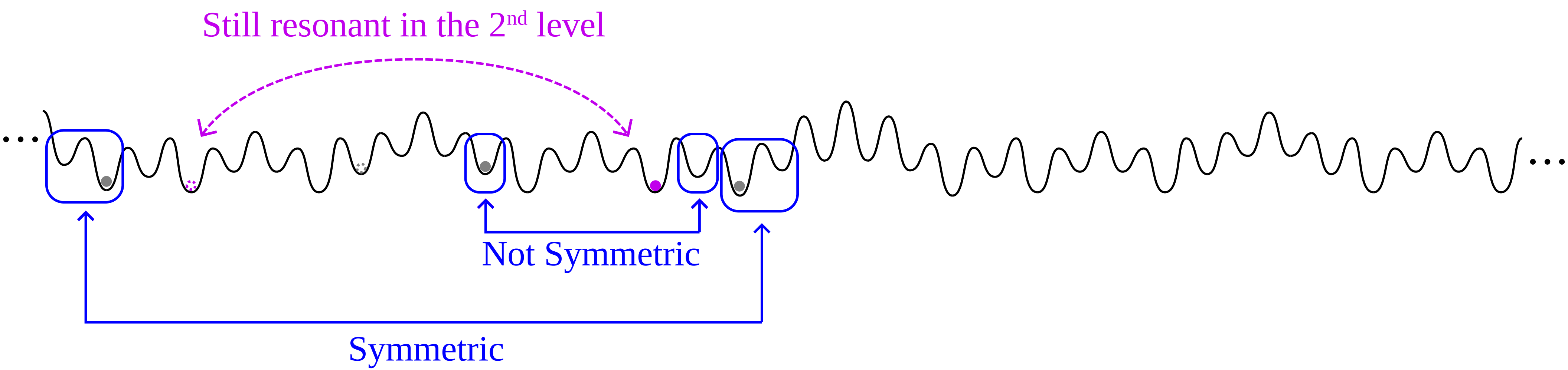

As long as there are resonances at level n for arbitrarily large n,
the particle will go arbitrarily far away.

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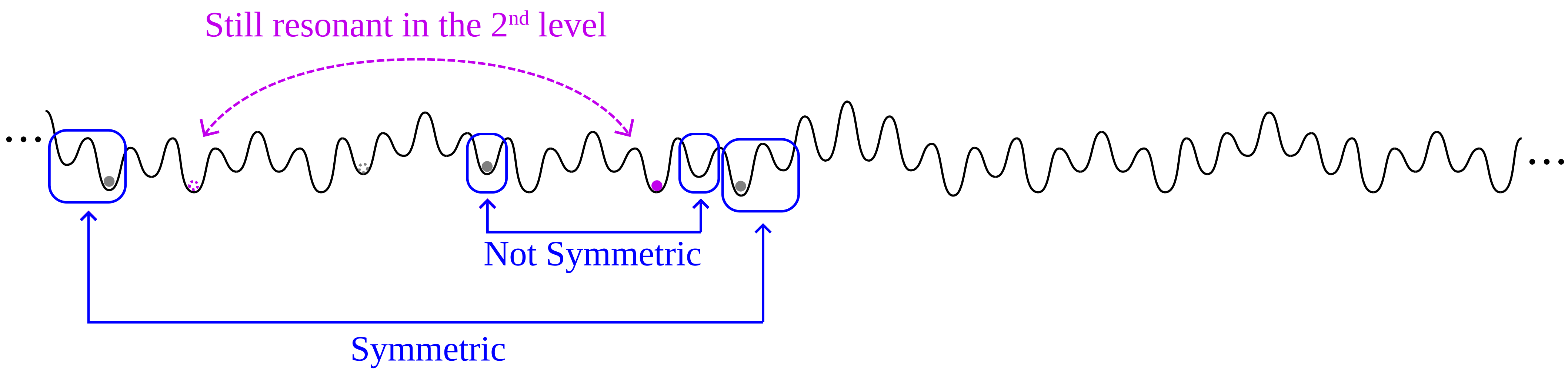
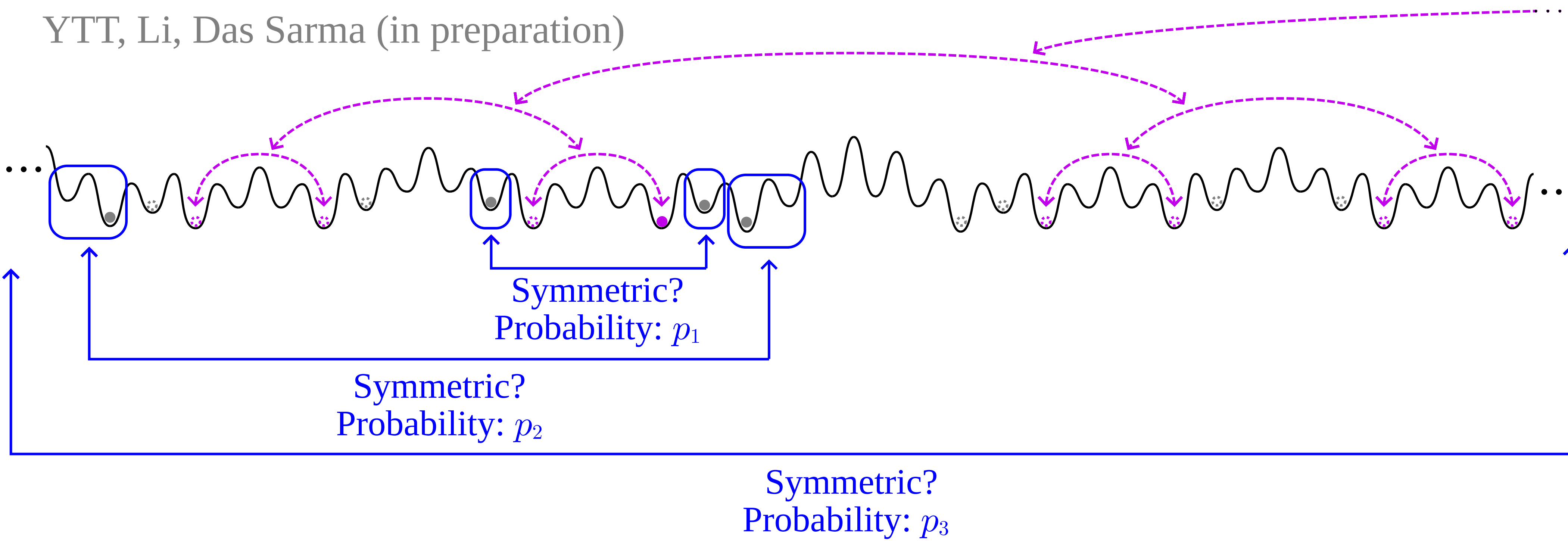


As long as there are resonances at level n for arbitrarily large n,
the particle will go arbitrarily far away.

(No need to resonate at every n)

Condition of delocalization in HMS with interactions

YTT, Li, Das Sarma (in preparation)



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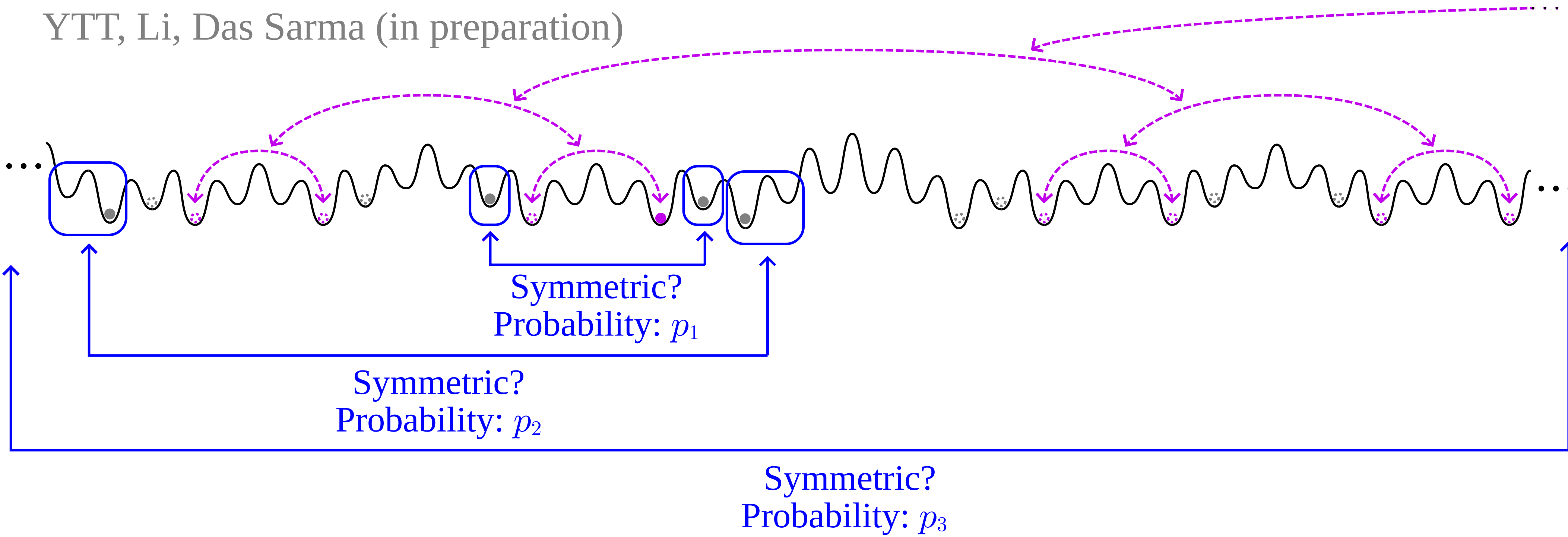

Expected number of resonances (0 or 1) at level n: p_n

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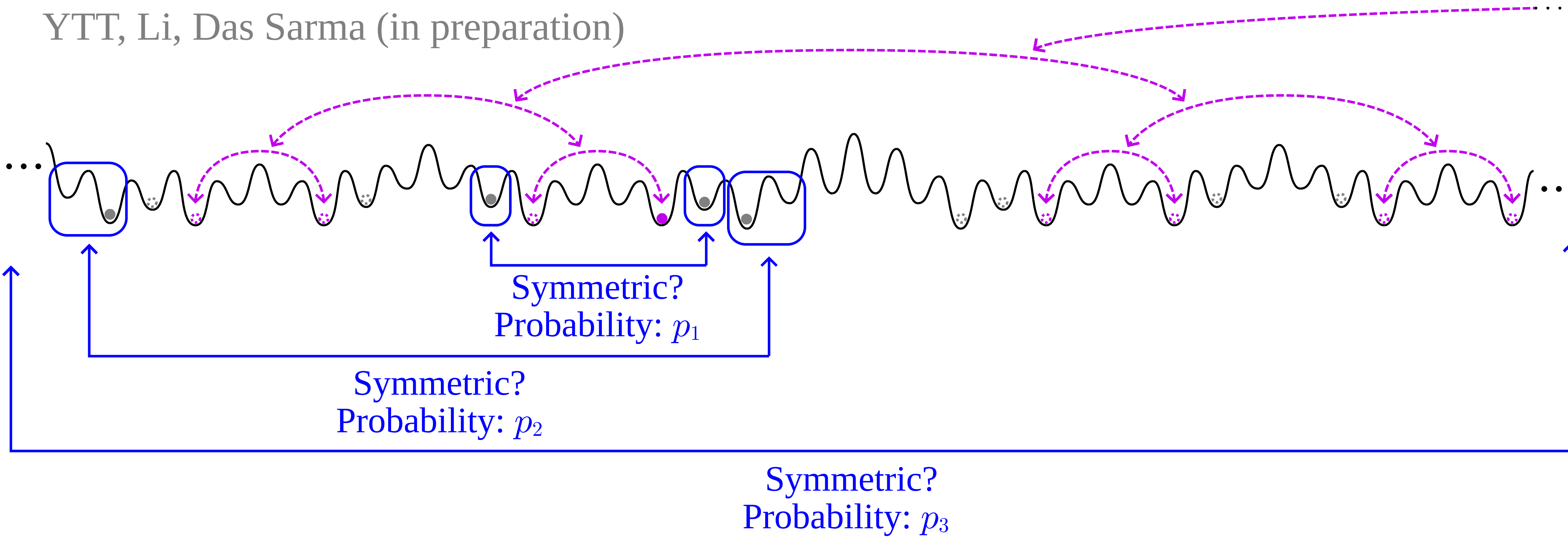

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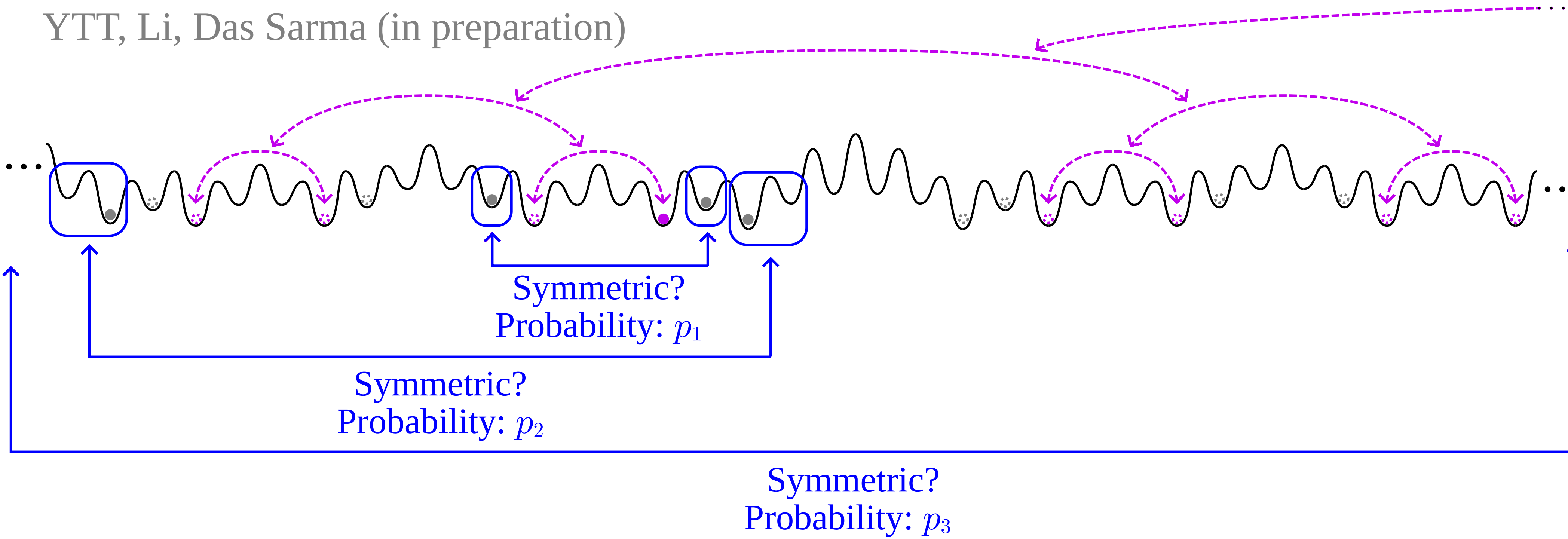

Expected number of resonances (0 or 1) at level n: p_n

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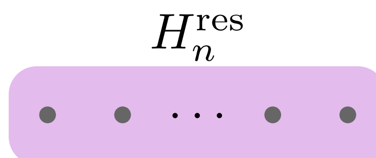
Taking $n \to \infty$, we get:

The particle goes arbitrarily far away iff $\Sigma\, p_n$ diverges.

Outline

- Introduction
  - Many-body localization (MBL)
  - Random v.s. Quasiperiodic systems
- Non-ergodic extended (NEE) regime
  - Numerical results
  - Theoretical explanation
- Many-body critical (MBC) phases
  - Single-particle hierarchical mirror structure (HMS)
  - Effects of interaction on HMS
  - Asymptotically solvable model

Construction of the solvable model of MBC



Random mixed-field Ising chain

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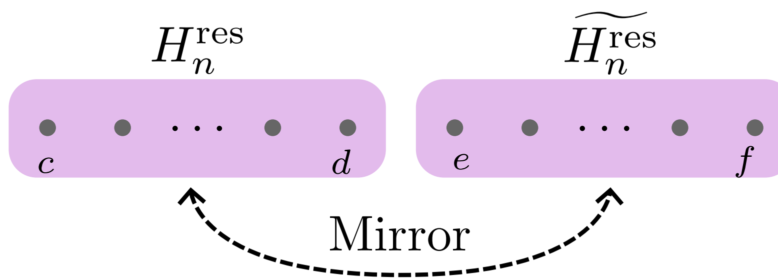
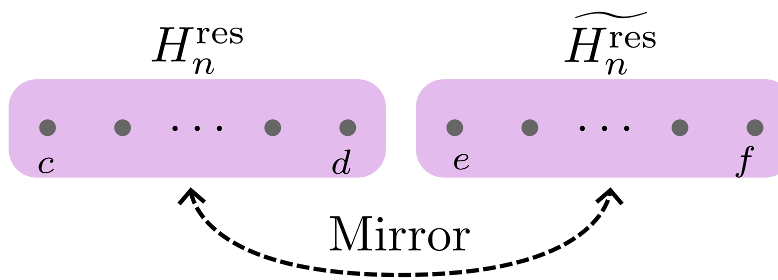


Construction of the solvable model of MBC

$\widetilde{X}$: mirror reflection of X



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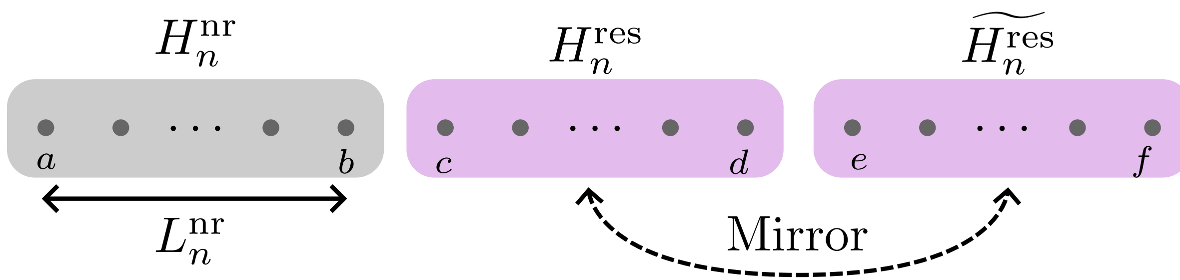

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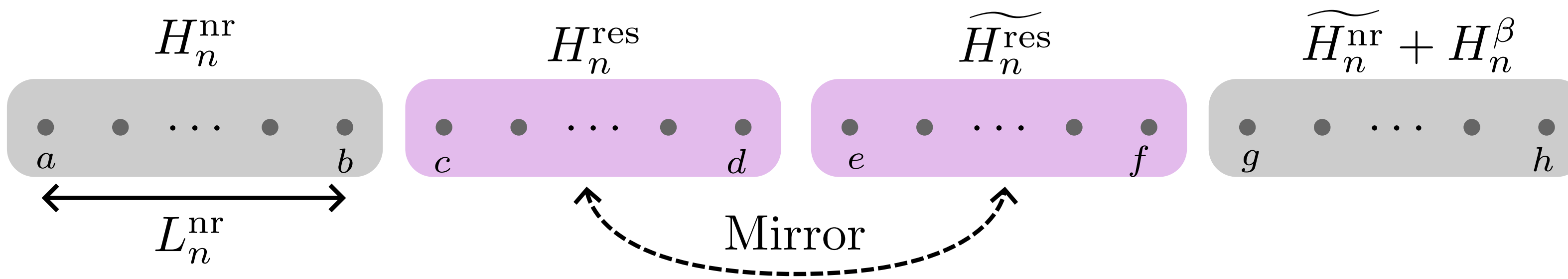

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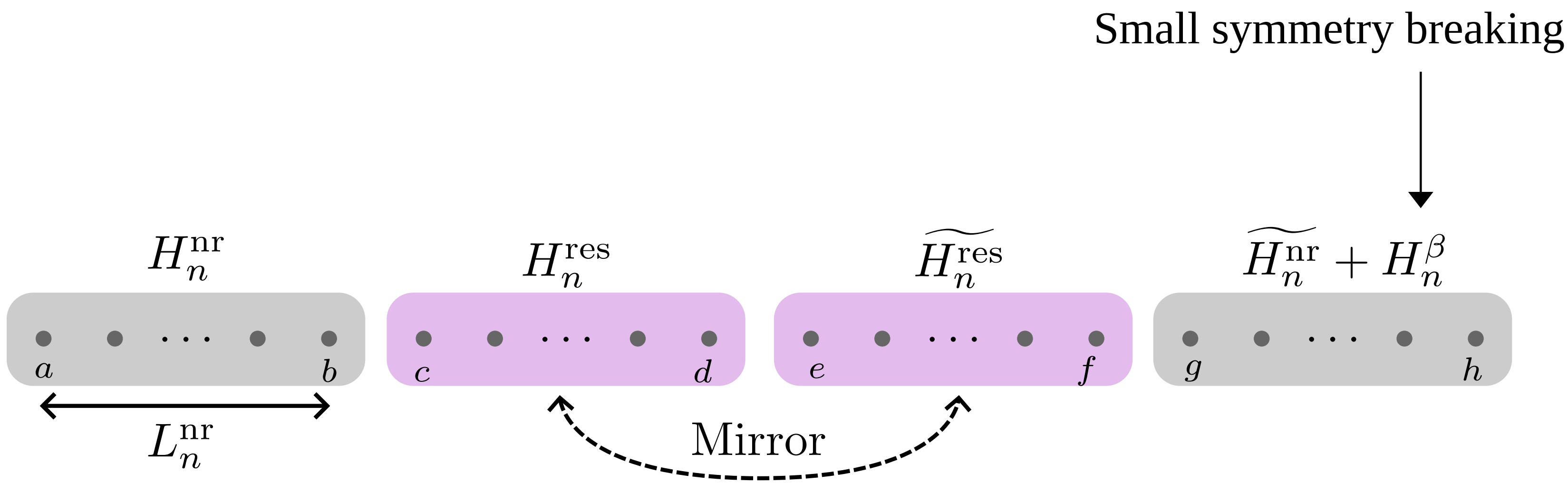

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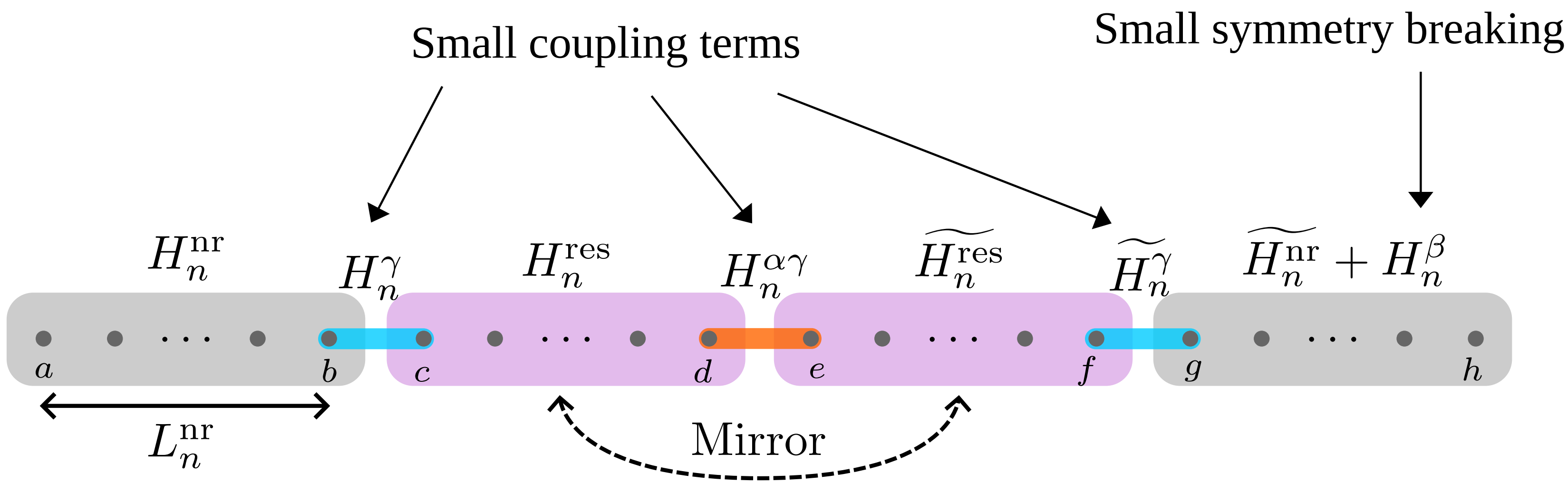

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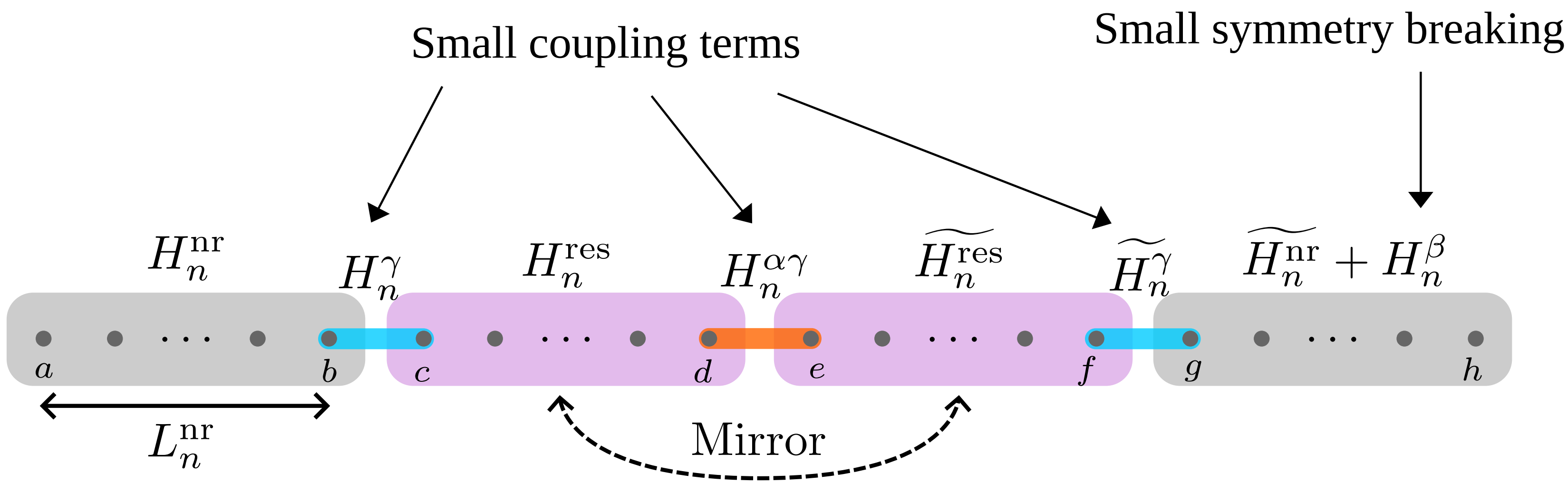

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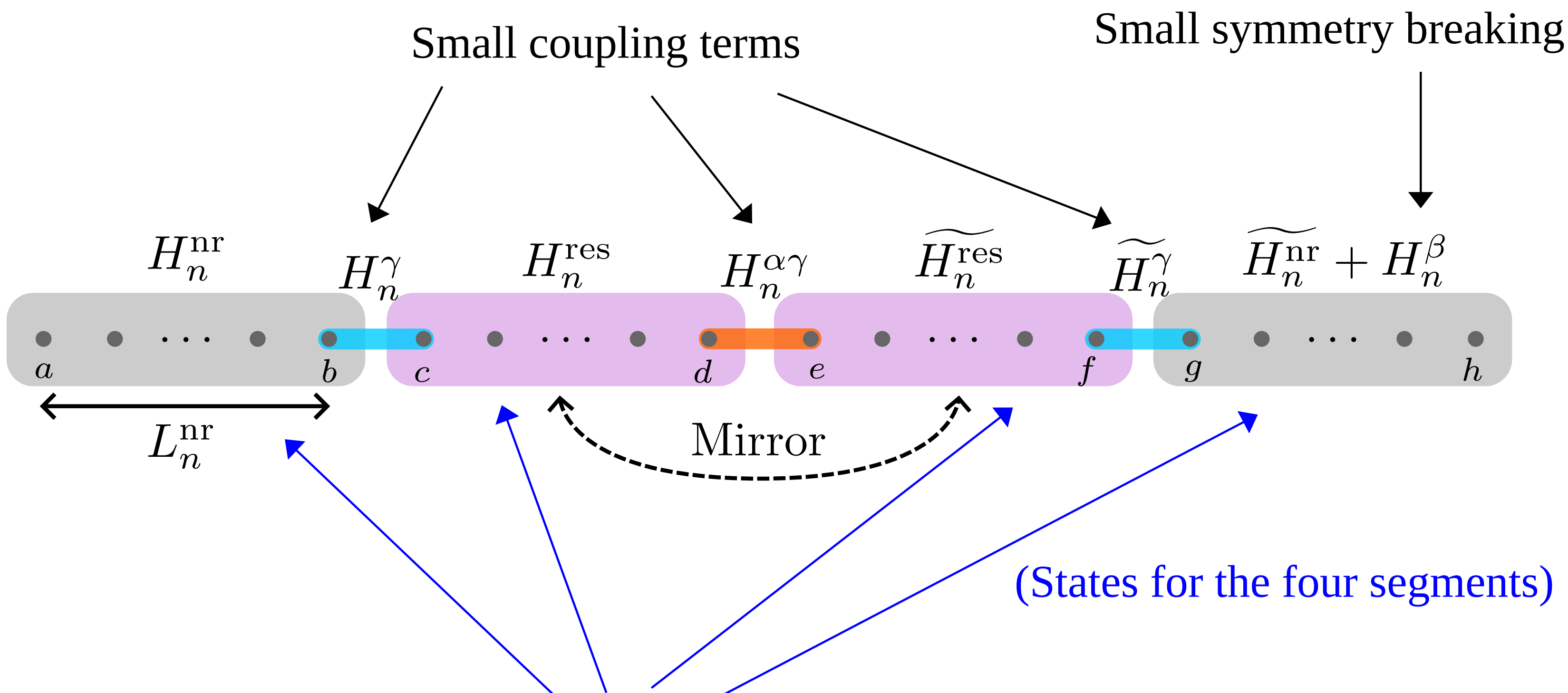

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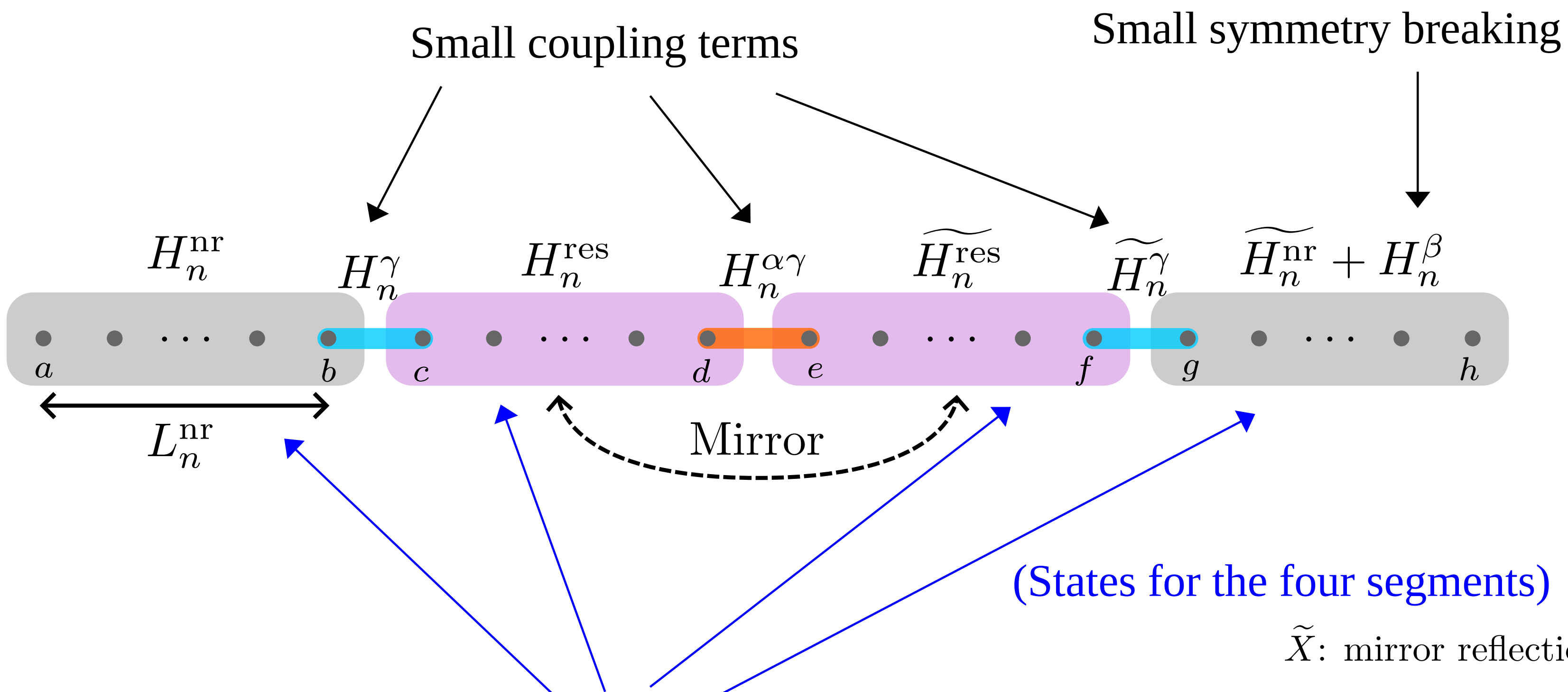
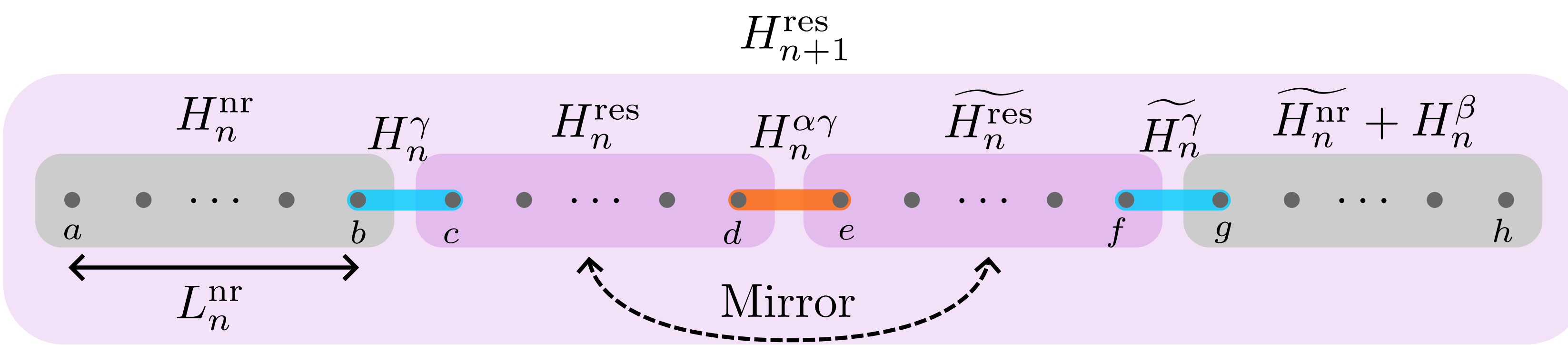


$\widetilde{X}$: mirror reflection of X

$$\text{perturbed eigenstate} \approx \begin{cases} |ABCD\rangle_n, & \text{if } A \neq \widetilde{D} \text{ or } B = C, \\ \frac{1}{\sqrt{2}}\left(|ABCD\rangle_n + |A\widetilde{C}\widetilde{B}D\rangle_n\right), & \text{if } A = \widetilde{D}, B > C, \\ \frac{1}{\sqrt{2}}\left(|ABCD\rangle_n - |A\widetilde{C}\widetilde{B}D\rangle_n\right), & \text{if } A = \widetilde{D}, B < C. \end{cases}$$

Construction of the solvable model of MBC



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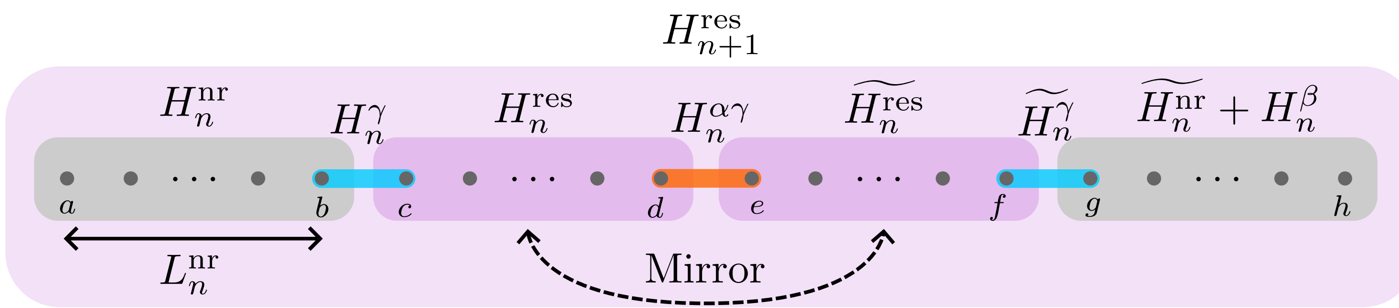

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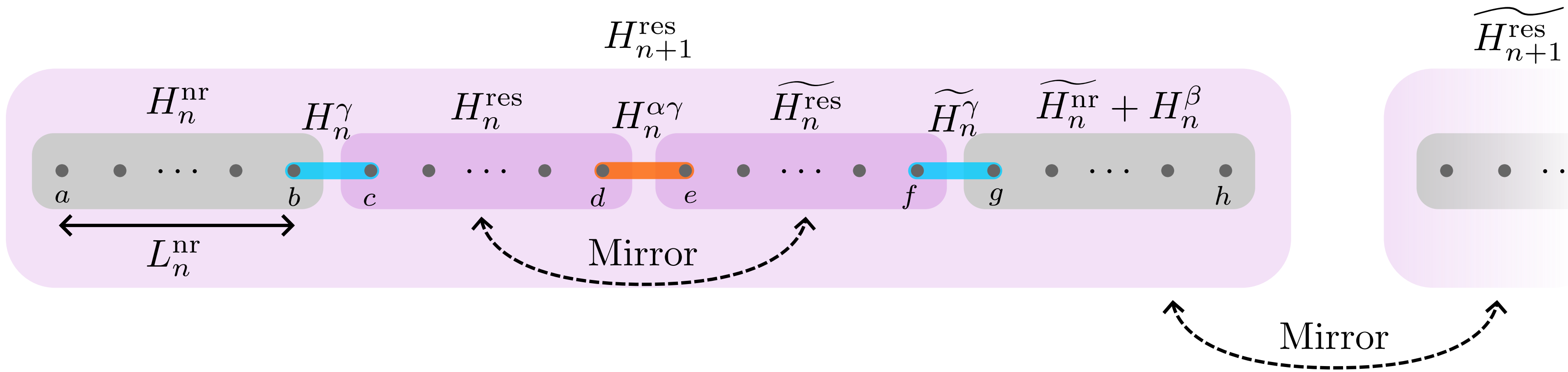

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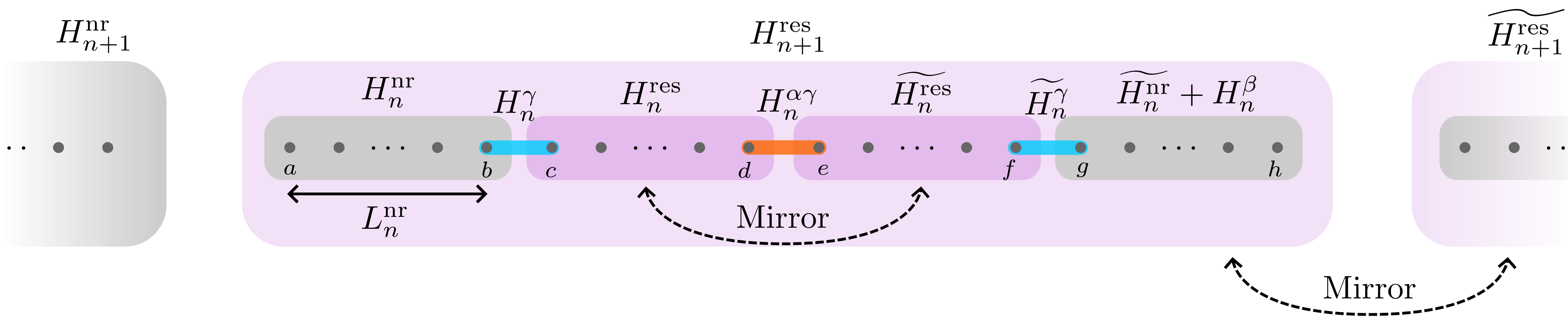

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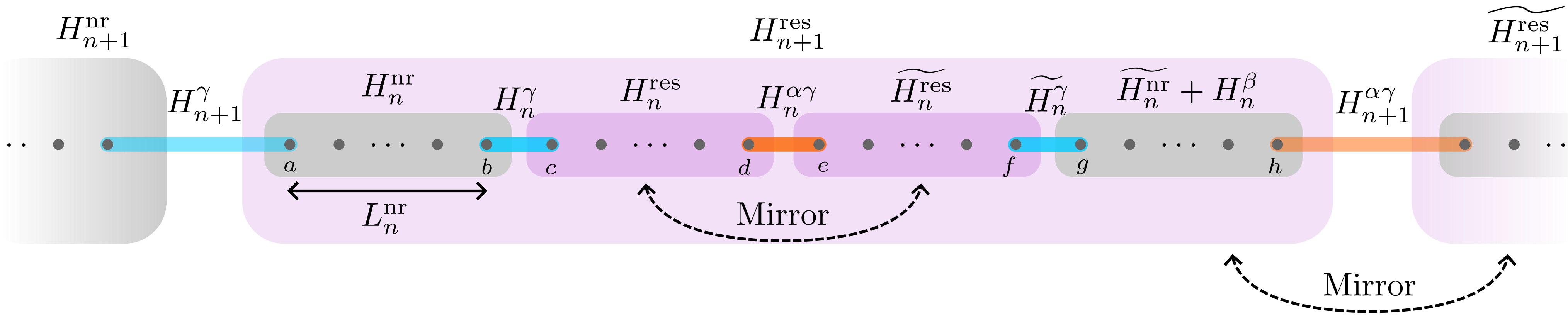

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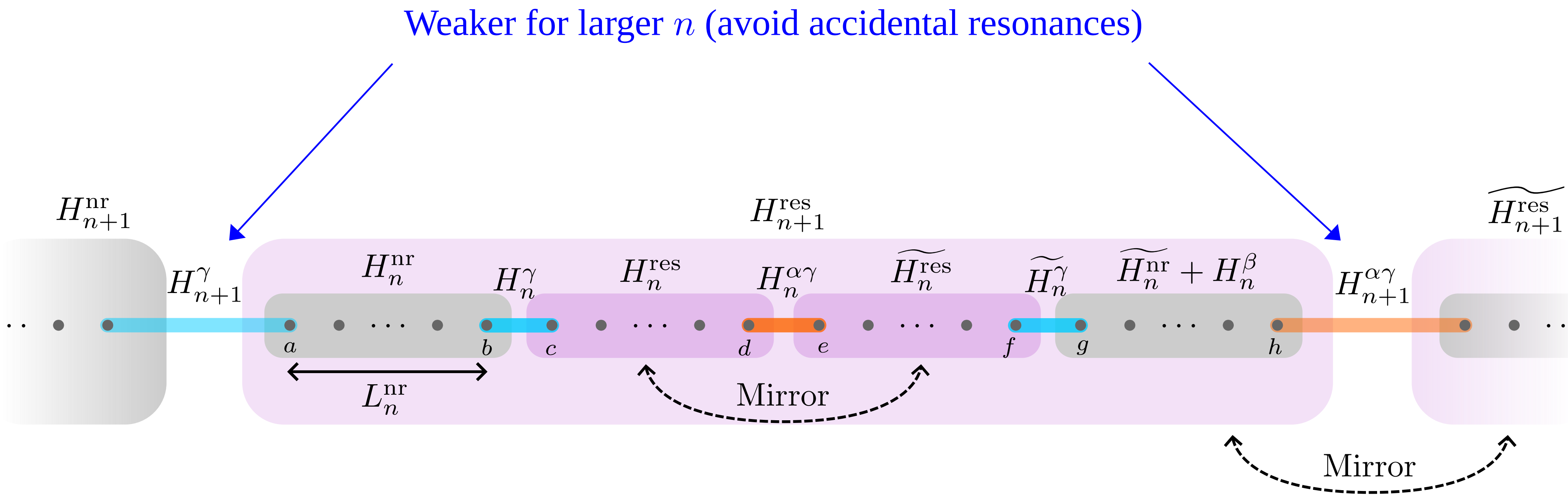

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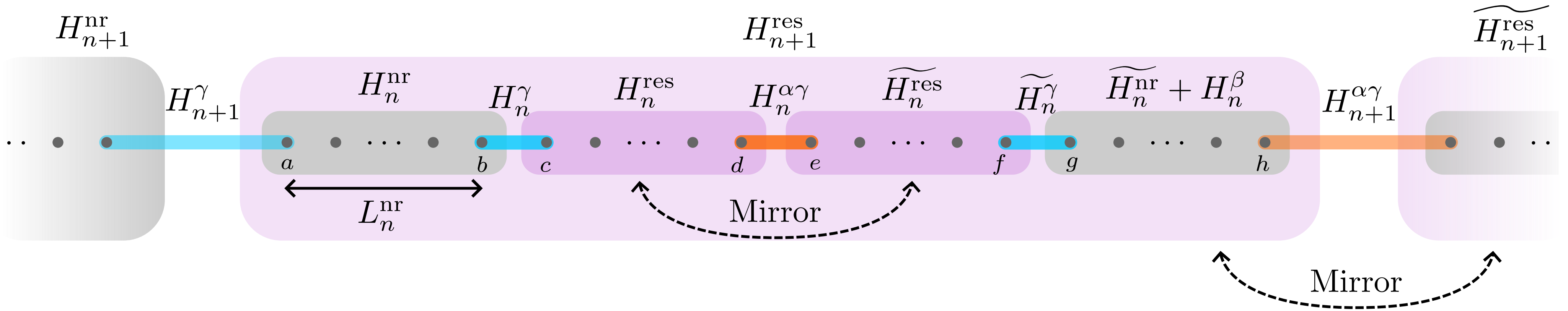
Probability of mirror resonances



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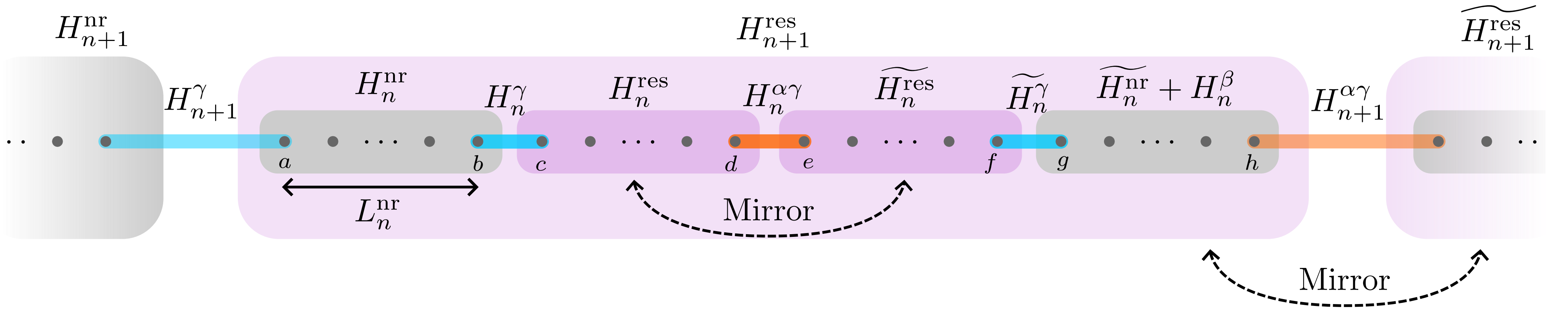

Randomly choose a state from the product canonical ensemble of the blocks at level n:

$$|ABCD\rangle_n \sim \rho^{\mathrm{nr}}_n \otimes \rho^{\mathrm{res}}_n \otimes \widetilde{\rho^{\mathrm{res}}_n} \otimes \widetilde{\rho^{\mathrm{nr}}_n}$$

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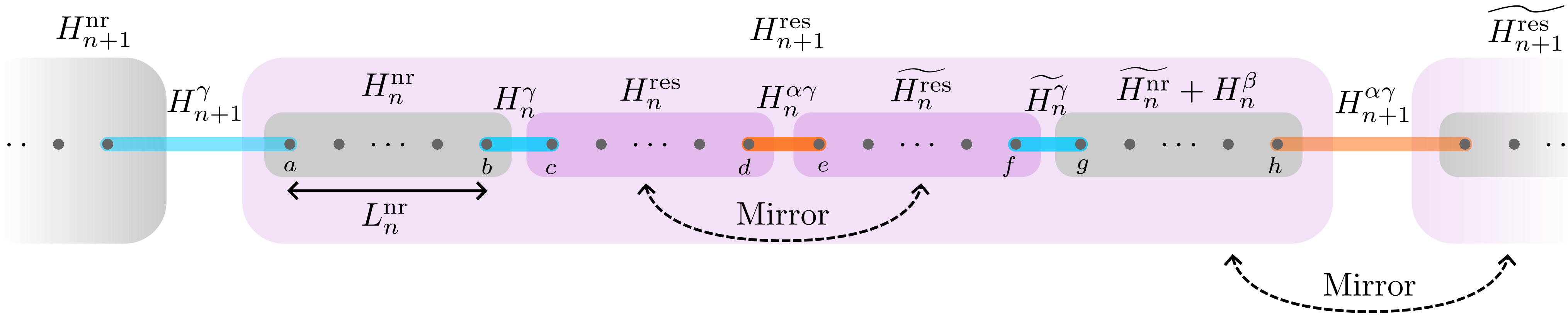

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The corresponding state at the next level becomes:

$$|ABCD\rangle_{n+1} \approx \begin{cases} |ABCD\rangle_n, & \text{with probability } 1-p_n \\ \frac{1}{\sqrt{2}}\big(|ABCD\rangle_n \pm |A\widetilde{C}\widetilde{B}D\rangle_n\big), & \text{with probability } p_n \end{cases}$$

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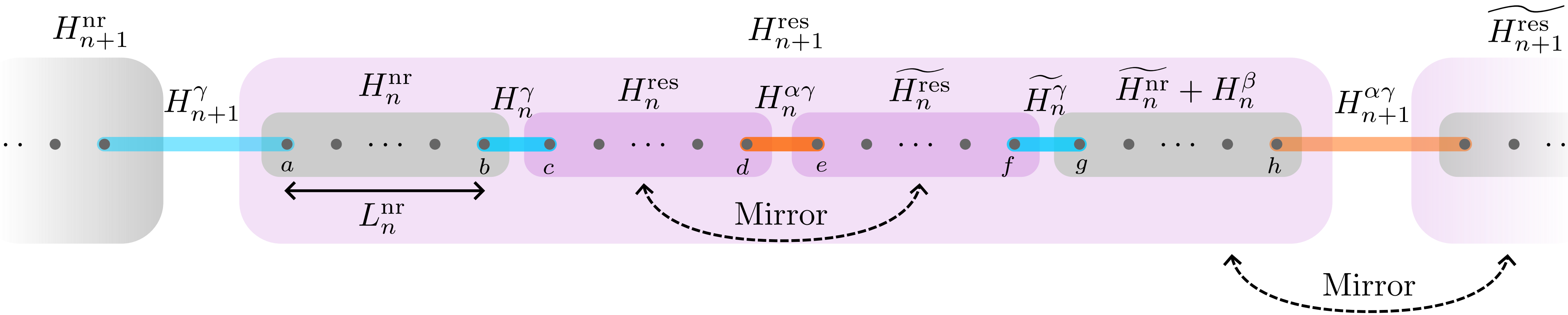

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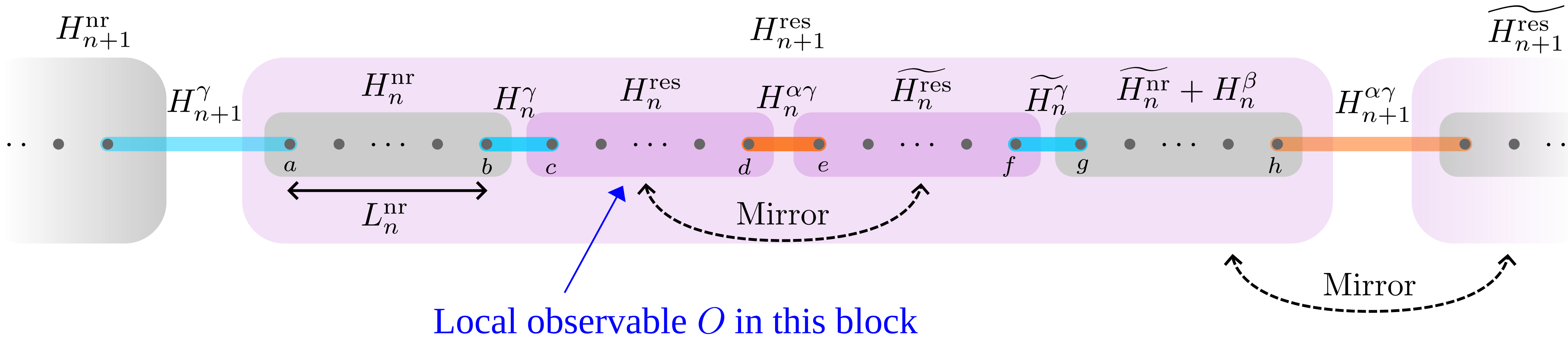
$$p_n = \mathrm{P}(A = \widetilde{D}) = \exp(-S^{\mathrm{nr}}_{2,n}), \quad S_2\text{: thermodynamic Rényi-2 entropy}$$

Variance of local observable expectations



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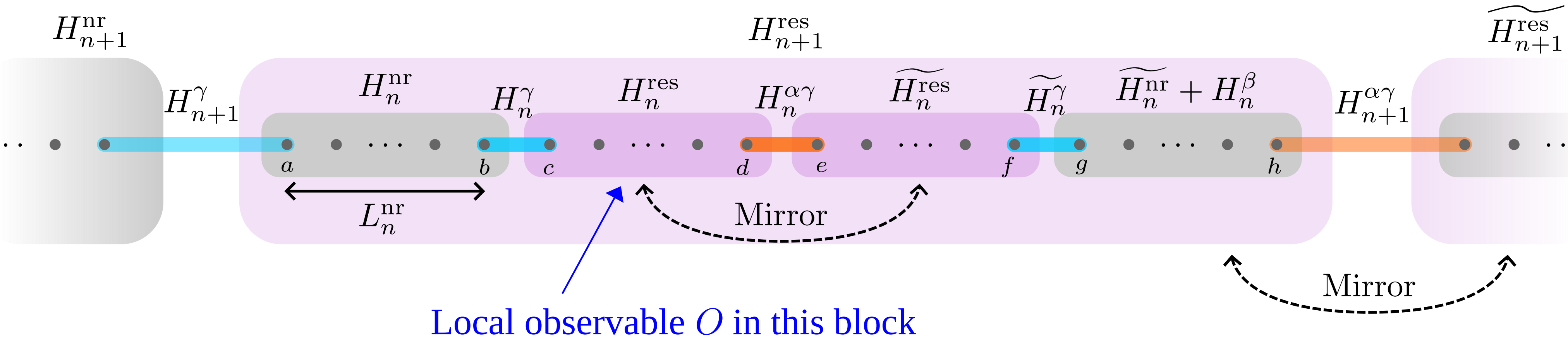

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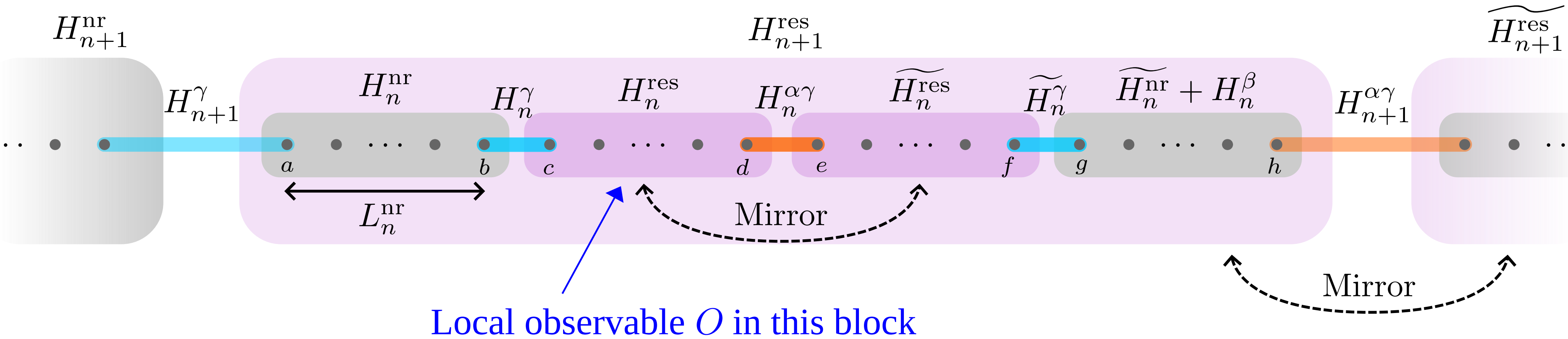

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$$\mathrm{Var}\langle X|O|X\rangle_{n+1} \approx (1-p_n)\,\mathrm{Var}\langle X|O|X\rangle_n + p_n \frac{\mathrm{Var}\langle X|O|X\rangle_n + \mathrm{Var}\langle \widetilde{X}|O|\widetilde{X}\rangle_n}{4}$$

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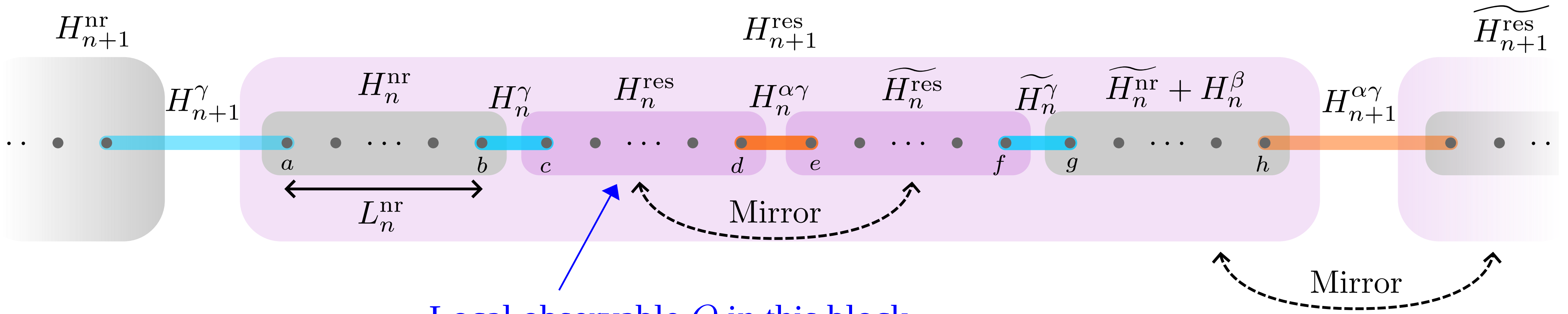

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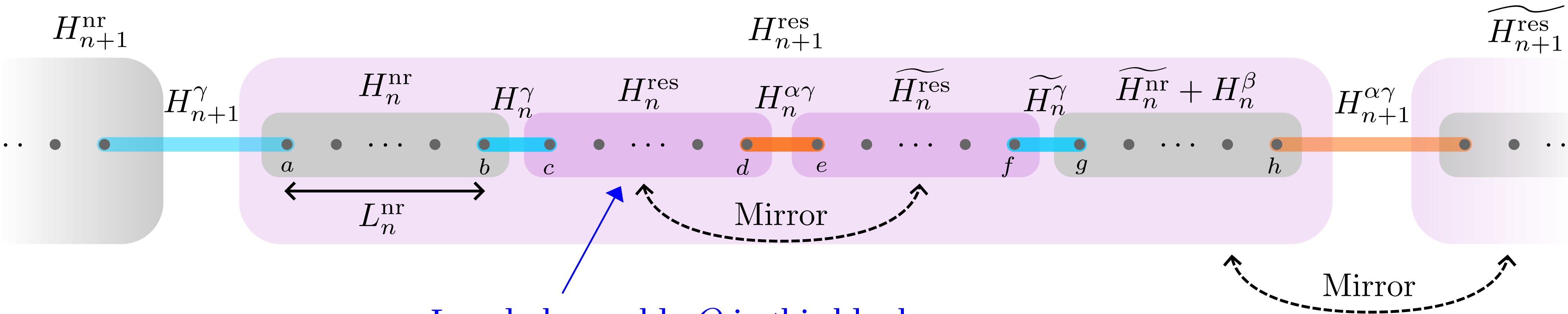

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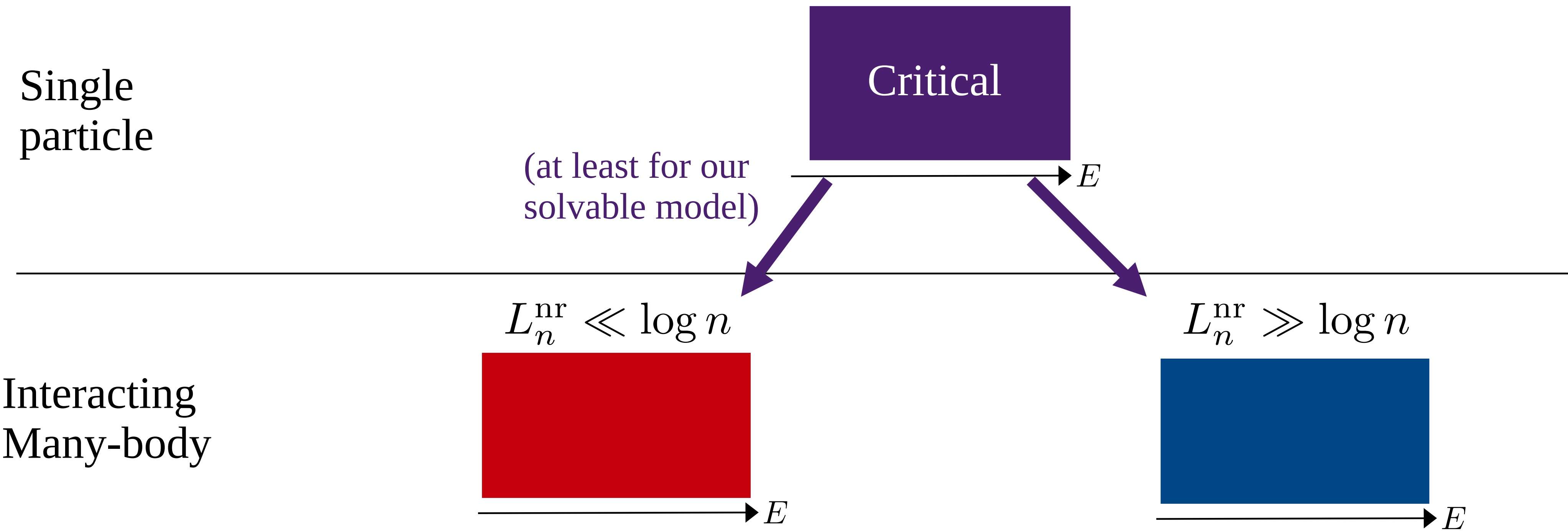
$$\boxed{\mathrm{Var}\langle X|O|X\rangle_n \to 0 \text{ (weak ETH)} \iff \textstyle\sum_n p_n = \infty}$$

Many-body critical phases

YTT, Li, Das Sarma (in preparation)



ETH-like states

$$\langle X|O|X\rangle_n \to \langle O\rangle_{\mathrm{thermal}}$$

MBL-like states

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Single
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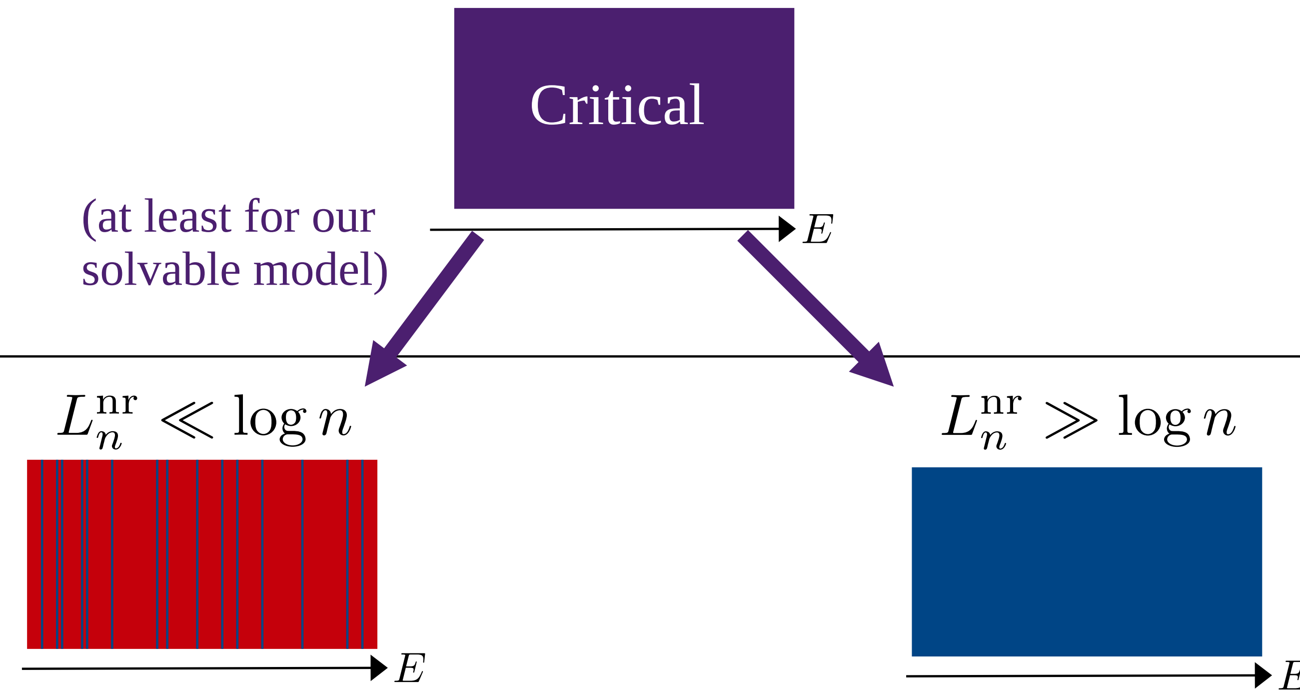

Interacting
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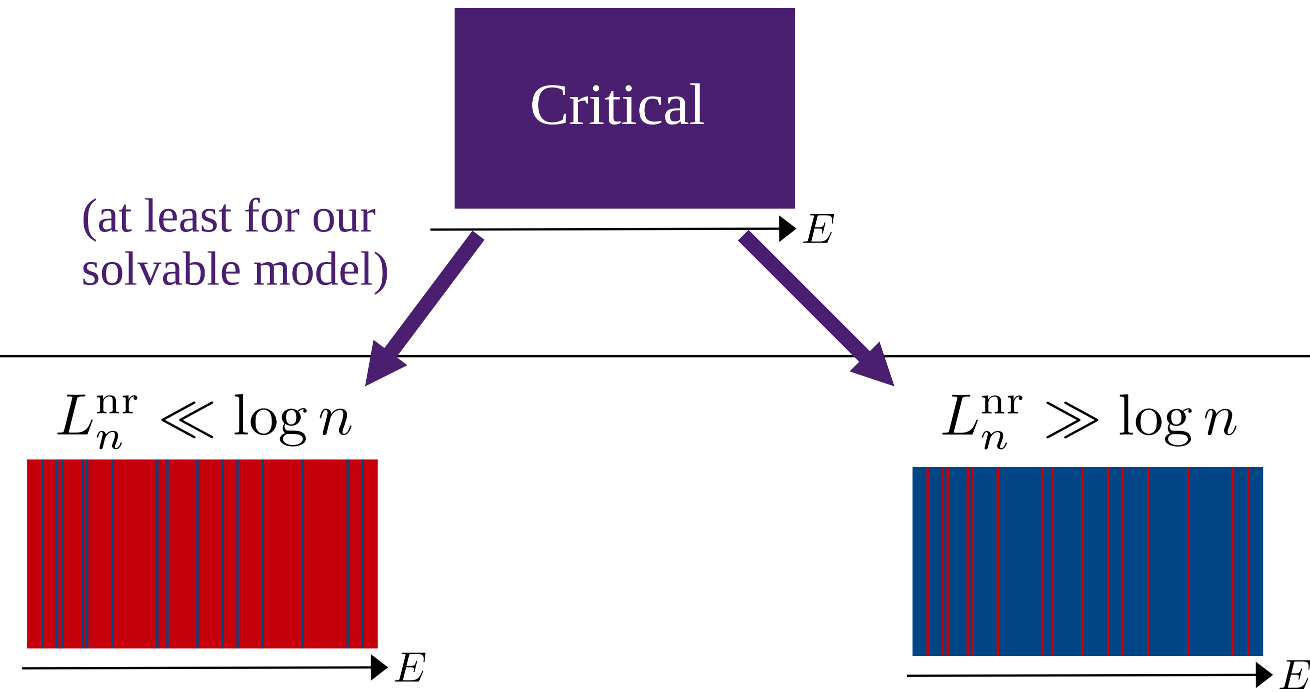

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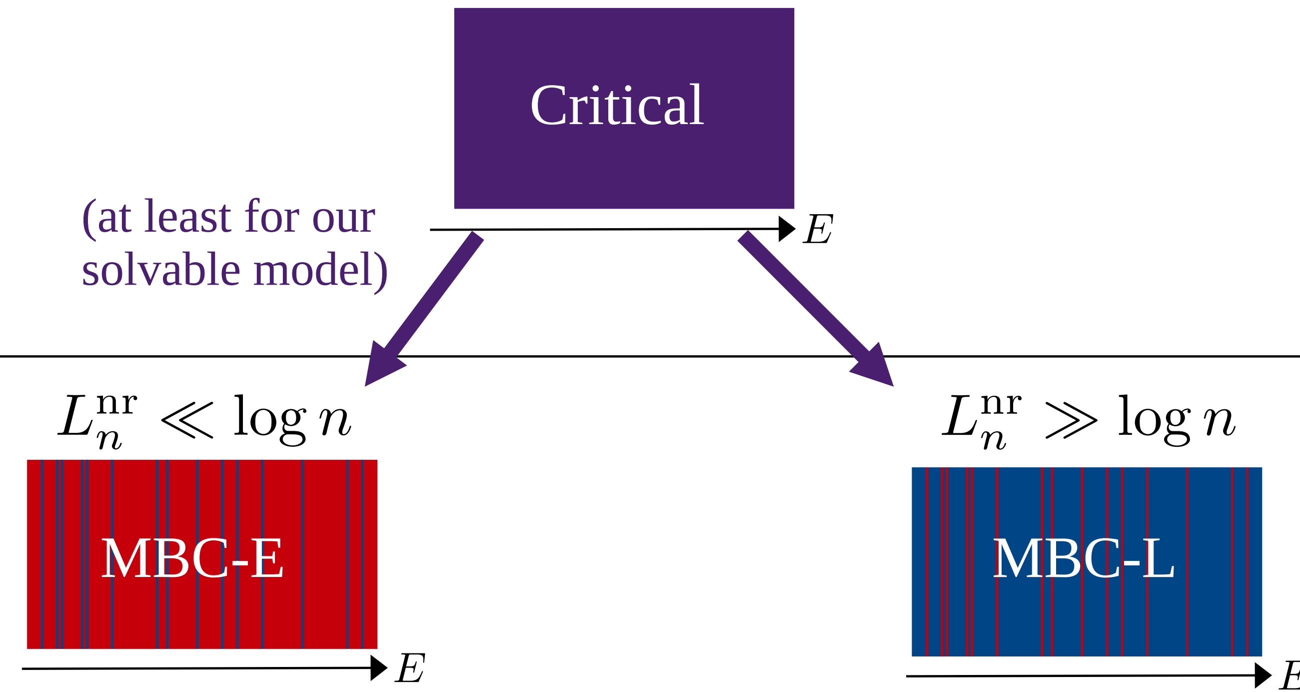

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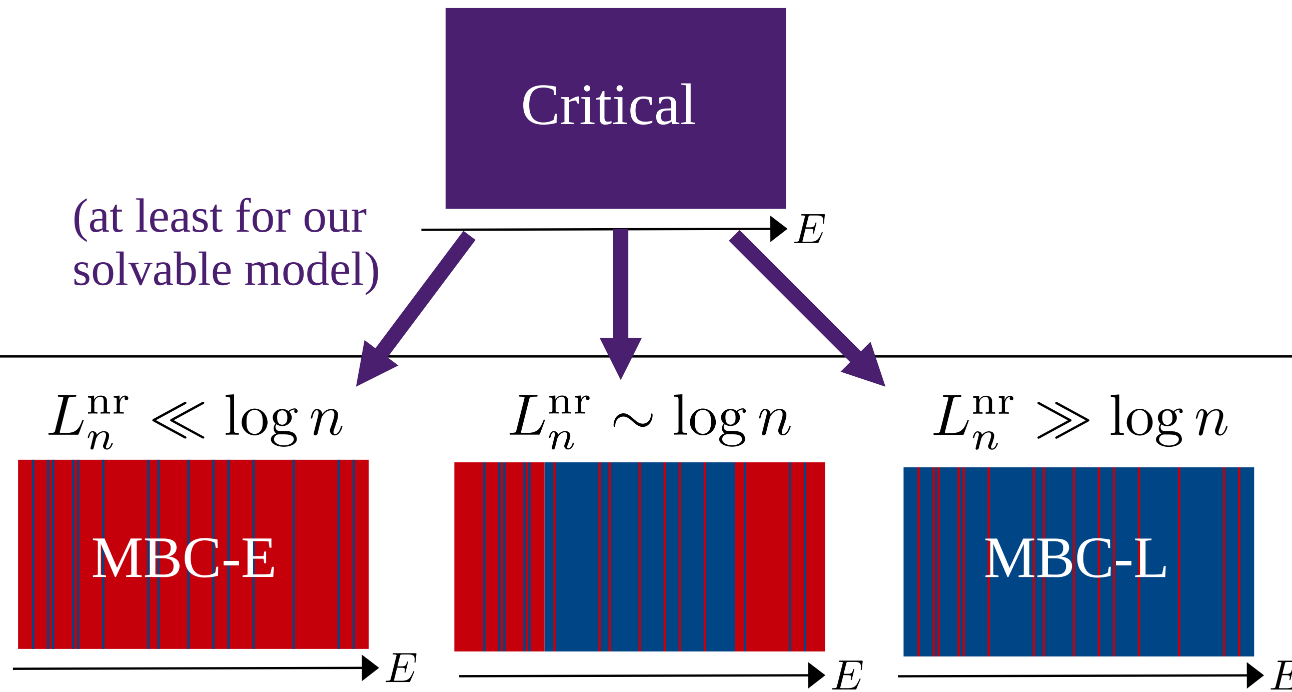

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Conclusions & general picture

For a quasiperiodic spin chain, we expect:

System size
or time scale

$L \sim 20$

$L \gtrsim 100$

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NEE behavior happens
When there are regularly-
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some quantities but not
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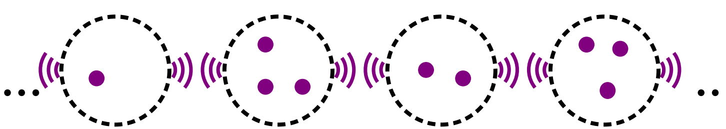

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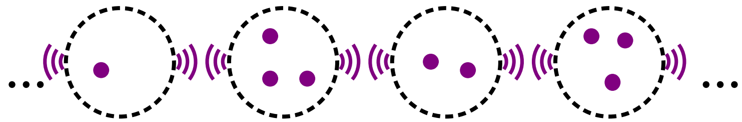

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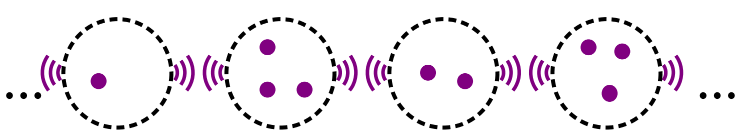

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Influenced by
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Hierarchical mirror structure can lead to:
- MBC-L phase (most states are MBL)
- MBC-E phase (most states are ETH)
generalizing the single-particle critical phase